

MARK BOX		
PROBLEM	POINTS	
1 a-j	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
%	100	

NAME: _____

please check the box of your section

Section 001 (MW 9:05 am)

or

Section 002 (MW 10:10 am)

INSTRUCTIONS:

- (1) To receive credit you must:
 - (a) **work in a logical fashion, show all your work, indicate your reasoning;**
no credit will be given for an answer that *just appears*;
such explanations help with partial credit
 - (b) if a line/box is provided, then:
 - show you work BELOW the line/box
 - put your answer on/in the line/box
 - (c) if no such line/box is provided, then box your answer
- (2) The MARK BOX indicates the problems along with their points.
Check that your copy of the exam has all of the problems.
- (3) You may **not** use a calculator, books, personal notes.
- (4) During this exam, do not leave your seat. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.
- (5) This exam covers (from *Calculus* by Anton, Bivens, Davis 8th ed.):
Sections 8.1, 8.2, 8.3, 8.4, 8.5, 8.7, 8.8. .

Problem Inspiration: If I told you here, you would know what method to use. So see the solution key, which will be available from the course homepage shortly after the exam.

Hints:

- (1) **You can check your answers to the indefinite integrals by differentiating.**
- (2) **+ C**
- (3) **For more partial credit, box your $u - du$ substitutions.**

1. Fill in the blanks (each worth 1 point).

1a. If a is a constant and $a > 0$ but $a \neq 1$, then

$$\int a^u du = \underline{\hspace{10cm}} + C$$

1b. $\int \tan u du = \underline{\hspace{10cm}} + C$

1c. If a is a constant and $a > 0$ then

$$\int \frac{1}{a^2+u^2} du = \underline{\hspace{10cm}} + C$$

1d. Partial Fraction Decomposition. If one wants to integrate $\frac{f(x)}{g(x)}$ where f and g are polynomials and $[\text{degree of } f] \geq [\text{degree of } g]$, then one must first do $\underline{\hspace{10cm}}$

1e. Integration by parts formula: $\int u dv = \underline{\hspace{10cm}}$

1f. Trig substitution: (recall that the *integrand* is the function you are integrating)
if the integrand involves $a^2 - u^2$, then one makes the substitution $u = \underline{\hspace{10cm}}$

1g. Trig substitution:
if the integrand involves $a^2 + u^2$, then one makes the substitution $u = \underline{\hspace{10cm}}$

1h. Trig substitution:
if the integrand involves $u^2 - a^2$, then one makes the substitution $u = \underline{\hspace{10cm}}$

1i. If you numerically approximate $\int_{x=a}^{x=b} f(x) dx$ using the Trapezoid Rule T_n , partitioning the interval $[a, b]$ into n subintervals, and if f'' is continuous on $[a, b]$, then

$$\left| T_n - \int_{x=a}^{x=b} f(x) dx \right| \leq \frac{(b-a)^3 K_2}{12n^2}$$

where

$$K_2 =$$

1j. If you numerically approximate $\int_{x=a}^{x=b} f(x) dx$ using Simpson's Rule S_{2n} , partitioning the interval $[a, b]$ into $2n$ subintervals, and if $f^{(4)}$ is continuous on $[a, b]$, then

$$\left| S_{2n} - \int_{x=a}^{x=b} f(x) dx \right| \leq \frac{(b-a)^5 K_4}{180(2n)^4}$$

where

$$K_4 =$$

2. Improper Integral. Use correct notation and verify your reasoning. Convince your reader that you understand what is coming on. A picture is worth 1000 words.

2a. Make a rough sketch of the graph of $y = \frac{1}{x}$ for $0 < x < \infty$.

2b.

$$\int_{x=0}^{x=17} \frac{1}{x} dx =$$

2c.

$$\int_{x=17}^{x=\infty} \frac{1}{x} dx =$$

2d.

$$\int_{x=0}^{x=\infty} \frac{1}{x} dx =$$

3. Let's numerically approximate

$$\int_{x=5}^{x=7} \sqrt{x} \, dx$$

by using **Simpson's Rule** S_{2n} , partitioning the interval $[5, 7]$ into $2n = 6$ subintervals.

3a.

$$S_6 \approx$$

3b. Following the notation from problem 1j of this exam, for problem 3

$$K_4 =$$

Hint: your answer should be a number and carry out the arithmetic as far as indicated in class.

3c. The error bound for Simpson's Rule is:

$$\left| S_6 - \int_{x=5}^{x=7} \sqrt{x} \, dx \right| \leq$$

Hint: your answer should be a number and carry out the arithmetic as far as indicated in class.

4.

$$\int \tan(17x) \sec^3(17x) dx =$$

+ C

5.

$$\int \frac{x^2}{\sqrt{4-x^2}} dx =$$

+ C

6.

$$\int \frac{x}{x^4 + 4x^2 + 8} dx =$$

+ C

7.

$$\int \frac{5x^3 - 3x^2 + 2x - 1}{x^4 + x^2} dx =$$

+ C

8.

$$\int t e^{-17t} dt =$$

+ C

9. LaPlace Transform (from a homework problem)

Consider a function of t , denoted by $y = f(t)$. The **LaPlace Transform** of this function $y = f(t)$ is a (new) function, namely the function

$$y = \mathcal{L}\{f(t)\} (s) ,$$

which is a function of s for $s > 0$. The formula for the LaPlace Transform of $y = f(t)$ is

$$\mathcal{L}\{f(t)\} (s) = \int_{t=0}^{t=\infty} e^{-st} f(t) dt \tag{9}$$

where, in the integral in (9) above, s is treated as a constant.

The LaPlace Transform of the function

$$f(t) = t$$

is the function

$\mathcal{L}\{f(t)\} (s) =$

for $s > 0$. Hint: your answer should be a function of s and problem **8** of this exam might help.

10. Let $n \geq 2$. Derive a reduction formula for $\int \sec^n x \, dx$.

$$\int \sec^n x \, dx =$$

Show your work, justifying your answer. If you memorized the formula and just write it down and do not show your justification of the formula, you will not receive any points.