Math 142

In-Class Quiz from Thursday 2/9/06

Due at beginning of class on Monday 2/13/06

The class performance on Problem 1 from Exam 1 was **absolutely unacceptable**: the average score was only 5.6 and the median was only 5 (out of 10 points). You were told that the first problem would be fill-in the blanks from the formulas on the **Math 141 Handout** that I handed out the first day of class. Thus here is a quiz to reinforce some of formulas.

Using only

(i) simple $u - du$ substitution

(ii) basic trig identities such as

$$\tan x \;=\; \frac{\sin x}{\cos x} \qquad \cot x \;=\; \frac{\cos x}{\sin x} \qquad \sec x \;=\; \frac{1}{\cos x} \qquad \csc x \;=\; \frac{1}{\sin x} \tag{A}$$

(iii) the fact that $y = e^x$ and $y = \ln x$ are inverse functions of eachother and so

$$a^x \;=\; e^{\ln(a^x)} \;=\; e^{(x \ln a)} \tag{B}$$

where a is a constant with $a > 0$ and $a \neq 1$

(iv) If $z > 0$ then

$$\ln \frac{1}{z} \;=\; \ln 1 \;-\; \ln z \;=\; -\; \ln z \tag{C}$$

derive the formulas listed in (1) through (5) below. These formulas are from your Math 141 Handout. Derive means to show that they *easily* follow from (i) through (iv) above.

So if you ever forget the below formulas on an exam again, you will know how to easily and quickly derive them.

---

(1) If a is a constant and $a > 0$ but $a \neq 1$, then

$$\int a^x \, dx \;=\; \frac{a^x}{\ln a} + C \;.$$

Hint: use (B) above (more than once) and let $u = x \ln a$.

(2)

$$\int \tan x \, dx \;=\; -\,\ln \mid \cos x \mid + C$$

$$\int \tan x \, dx \;=\; \ln \mid \sec x \mid + C$$

Hints: use (A) and (C) above and let $u = \cos x$.

(3)

$$\int \cot x \, dx \;=\; \ln \mid \sin x \mid + C$$

$$\int \cot x \, dx \;=\; -\,\ln \mid \csc x \mid + C$$

Hints: use (A) and (C) above and let $u = \sin x$.

(4)

$$\int \sec x \, dx \;=\; \ln \mid \sec x + \tan x \mid + C$$

$$\int \sec x \, dx \;=\; -\,\ln \mid \sec x - \tan x \mid + C$$

$$\int \sec x \, dx \;=\; -\,\ln \mid \tan x - \sec x \mid + C$$

Hints.

First multiply the integrand through by the number one, written in a clever way:

$$\frac{\sec x + \tan x}{\sec x + \tan x} \quad \text{or} \quad \frac{\sec x - \tan x}{\sec x - \tan x} \quad \text{or} \quad \frac{\tan x - \sec x}{\tan x - \sec x} \; .$$

Then let u be either:

$$\sec x + \tan x \quad \text{or} \quad \sec x - \tan x \quad \text{or} \quad \tan x - \sec x \; .$$

(5)

$$\int \csc x \, dx \;=\; \ln \mid \csc x - \cot x \mid + C$$

$$\int \csc x \, dx \;=\; \ln \mid \cot x - \csc x \mid + C$$

$$\int \csc x \, dx \;=\; -\,\ln \mid \csc x + \cot x \mid + C$$

Hints.

First multiply the integrand through by the number one, written in a clever way:

$$\frac{\csc x - \cot x}{\csc x - \cot x} \quad \text{or} \quad \frac{\cot x - \csc x}{\cot x - \csc x} \quad \text{or} \quad \frac{\csc x + \cot x}{\csc x + \cot x} \; .$$

Then let u be either:

$$\csc x - \cot x \quad \text{or} \quad \cot x - \csc x \quad \text{or} \quad \csc x + \cot x \; .$$