| MARK BOX | | |
|---|---|---|
| PROBLEM | POINTS | |
| 1 a – y | 25 | in class |
| 2 a – o | 15 | in class |
| 3 | 10 | in class |
| 4 – 16 (10 pts each) | 130 | in class |
| 17 a–f **take home** | 10 | |
| 18 **take home** | 10 | |
| TOTAL | 200 | |

**NAME:** ______________________________

**please check the box of your section**

☐ **Section 005 (WF 8:00 am)**

or

☐ **Section 006 (WF 9:05 am)**

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The (whole) final exam covers (from *Calculus* by Anton, Bivens, Davis $8^{\text{th}}$ ed.):
Sections: 7.1 - 7.4, 7,6, 7.7, 8.1 - 8.5, 8.8, 10.1 - 10.10 .

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The above **MARK BOX** and below **Problem Inspiration** gives you an idea of the break down.
**Problem Inspiration**:

**IN CLASS: 1.–16.**

**1.–3.** Fill in the blanks and True/False (omitting sections 10.7, 10.9, 10.10)
**4.&5.** Chapter 7
**6.–11.** Chapter 8
**12.–16** Sections 10.1 – 10.6 and 10.8

**TAKE HOME 17.–18.**

**17.&18** Sections 10.7, 10.9, 10.10

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**INSTRUCTIONS for TAKE HOME PART**, which is due at 2pm on December 15, 2006:

(1) **Turn in all 5 pages of this exam.**
(2) To receive credit you must:
   (a) **work in a logical fashion, show all your work, indicate your reasoning**; **no credit will be given for an answer that *just appears***; such explanations help with partial credit
   (b) if a line/box is provided, then:
   — show you work BELOW the line/box
   — put your answer on/in the line/box
   (c) if no such line/box is provided, then box your answer
(3) You can use books and notes.
(4) **You can not use a calculator. You can not use a computer.** Thus you do not need to do lots of *multiplication* and may leave you answers as you would on previous exams (e.g. $\frac{(7)(17)}{5!}$ is acceptable).
(5) **You can not receive help from other people.**

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**SIGNATURE REQUIRED:**

I hereby verify that I did NOT receive help from other people on this final exam take-home part.

**Signature:** ______________________________

Nothing to fill in on this page – Prof. Girardi just included it to help you out.

Let $y = f(x)$ be a function with derivatives of all orders in an interval $I$ containing $x_0$.
Let $y = P_N(x)$ be the $N^{\text{th}}$-order Taylor polynomial of $y = f(x)$ about $x_0$.
Let $y = R_N(x)$ be the $N^{\text{th}}$-order Taylor remainder of $y = f(x)$ about $x_0$.
Let $y = P_\infty(x)$ be the Taylor series of $y = f(x)$ about $x_0$.
Let $c_n$ be the $n^{\text{th}}$ Taylor coefficient of $y = f(x)$ about $x_0$.

**A.** In open form (i.e., with ... and without a $\sum$-sign)

$$P_N(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f^{(2)}(x_0)}{2!}(x-x_0)^2 + \frac{f^{(3)}(x_0)}{3!}(x-x_0)^3 + \cdots + \frac{f^{(N)}(x_0)}{N!}(x-x_0)^N$$

**B.** In closed form (i.e., with a $\sum$-sign and without ...)

$$P_N(x) = \sum_{n=0}^{N} \frac{f^{(n)}(x_0)}{n!}\ (x-x_0)^n$$

**C.** In open form (i.e., with ... and without a $\sum$-sign)

$$P_\infty(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f^{(2)}(x_0)}{2!}(x-x_0)^2 + \cdots + \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n + \ldots$$

**D.** In closed form (i.e., with a $\sum$-sign and without ...)

$$P_\infty(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!}\ (x-x_0)^n$$

**E.** We know that $f(x) = P_N(x) + R_N(x)$. Taylor's BIG Theorem tells us that, for each $x \in I$,

$$R_N(x) = \frac{f^{(N+1)}(c)}{(N+1)!}\ (x-x_0)^{(N+1)}$$

for some $c$ between $x$ and $x_0$ .

**F.** The formula for $c_n$ is

$$c_n = \frac{f^{(n)}(x_0)}{n!}$$

**17.** Do parts (a) - (f) for the following:

$$f(x) = xe^x \qquad x_0 = 0 \qquad J = (-17\ ,\ 2)$$

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You might find it easier to do problems (a) - (f) in a different order. Just do what you find easiest.

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Use only:

- the definition of Taylor polynominal
- the definition of Taylor series
- the theorem/error-estimate on the $N^{\text{th}}$-Remainder term for Taylor polynomials.

**Do NOT use a known Taylor Series (i.e., do not use methods from section 10.10).**

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**17a.** Find the following. Note the $1^{\text{st}}$ column are functions of $x$ and the $2^{\text{nd}}$ and $3^{\text{rd}}$ columns are numbers (do not get out a calculator and start pushing keys for the numbers).

| | | |
|---|---|---|
| $f^{(0)}(x) \ =$ | $f^{(0)}(x_0) \ =$ | $c_0 \ =$ |
| $f^{(1)}(x) \ =$ | $f^{(1)}(x_0) \ =$ | $c_1 \ =$ |
| $f^{(2)}(x) \ =$ | $f^{(2)}(x_0) \ =$ | $c_2 \ =$ |
| $f^{(3)}(x) \ =$ | $f^{(3)}(x_0) \ =$ | $c_3 \ =$ |
| $f^{(4)}(x) \ =$ | $f^{(4)}(x_0) \ =$ | $c_4 \ =$ |
| $f^{(5)}(x) \ =$ | nothing for here | nothing for here |

**17b.** Find the $N^{\text{th}}$-order Taylor polynomial of $y = f(x)$ about $x_0$ in OPEN form for $N = 0, 1, 2, 3, 4$.

| |
|---|
| $P_0(x) \ =$ |
| $P_1(x) \ =$ |
| $P_2(x) \ =$ |
| $P_3(x) \ =$ |
| $P_4(x) \ =$ |

**17c.** Find the Taylor series of $y = f(x)$ about $x_0$ in OPEN form.

$P_\infty(x) \;=$

**17d** Find the Taylor series of $y = f(x)$ about $x_0$ in CLOSED form.

$P_\infty(x) \;=$

**17e.** Consider the given interval $J$. Find an upper bound for the maximum of $\left|f^{(5)}(x)\right|$ on the interval $J$. You answer should be a number (leave it as a fraction - do not get out a calculator and start pushing keys). You answer cannot have an: $N$, $x$, $x_0$, $c$.

$\max_{c \in J} \left|f^{(5)}(c)\right| \;\le$

**17f.** Consider the given interval $J$. Using Taylor's Remainder Theorem (i.e., Taylor's Big Theorem), find an upper bound for the maximum of $|R_4(x)|$ on the interval $J$. You answer should be a number (leave it as a fraction - do not get out a calculator and start pushing keys). You answer cannot have an: $N$, $x$, $x_0$, $c$.

$\max_{x \in J} |R_4(x)| \;\le$

**18.** Using the fact that

$$\frac{1}{1-r} = \sum_{n=0}^{\infty} r^n \qquad \text{when} \qquad |r| < 1 \ , \tag{$*$}$$

find a power series expansion of

$$\frac{x}{4+100x^2}$$

and state when it is valid. Simplify your answer so that your power series has the form $\sum_{n=0}^{\infty} c_n x^{\text{some power}}$ for some constants $c_n$.

| |
|---|
| $\dfrac{x}{4+100x^2} = \displaystyle\sum_{n=0}^{\infty}$ valid when $\lvert x \rvert <$ |