MARK BOX		
PROBLEM	POINTS	
1 a – j	30	
2	5	
3	10	
4	10	
5 ab	10	
6	10	
7	10	
8	5	
9 - Part 1	10	
%	100	

NAME: ______________________________

please check the box of your section

☐ **Section 005 (WF 8:00 am)**

or

☐ **Section 006 (WF 9:05 am)**

INSTRUCTIONS:

(1) To receive credit you must:
  (a) **work in a logical fashion, show all your work, indicate your reasoning**; **no credit will be given for an answer that *just appears***; such explanations help with partial credit
  (b) if a line/box is provided, then:
  — show you work BELOW the line/box
  — put your answer on/in the line/box
  (c) if no such line/box is provided, then box your answer
(2) The MARK BOX indicates the problems along with their points. Check that your copy of the exam has all of the problems.
(3) You may **not** use a calculator, books, personal notes.
(4) During this exam, do not leave your seat. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.
(5) This exam covers (from *Calculus* by Anton, Bivens, Davis 8^{th} ed.):
Part 1: Sections 10.1, 10.2 and Part 2: Sections 10.3, 10.4, 10.5, 10.6 and 10.8 .

Problem Inspiration:

1. you were warned, from class handouts and old exams
2. homework problem § Ch 10 Review # 9 , homework problem § 10.4 # 28
3. Example from class lecture
4. Serious Series Problems # 10
5. homework problem § 10.6 # 29
6. homework problem § 10.8 # 29
7. from textbook § 10.8 # 49
8. homework problem § 10.8 # 63

1. Fill-in-the blanks/boxes. All series $\sum$ are understood to be $\sum_{n=1}^{\infty}$.

Hint: I do NOT want to see the words absolute nor conditional on this page!

1a. n^{th}**-term test** for an arbitrary series $\sum a_n$.
If $\lim_{n\to\infty} a_n \neq 0$ or $\lim_{n\to\infty} a_n$ does not exist, then $\sum a_n$ __________________ .

1b. Geometric Series where $-\infty < r < \infty$. The series $\sum r^n$

- converges if and only if $|r|$ __________________
- diverges if and only if $|r|$ __________________

1c. p**-series** where $0 < p < \infty$. The series $\sum \frac{1}{n^p}$

- converges if and only if p __________________
- diverges if and only if p __________________

1d. Integral Test for a positive-termed series $\sum a_n$ where $a_n \geq 0$.
Let $f\colon [1, \infty) \to \mathbb{R}$ be so that

- $a_n = f($_________$)$ for each $n \in \mathbb{N}$
- f is a ____________________________________ function
- f is a ____________________________________ function
- f is a ____________________________________ function .

Then $\sum a_n$ converges if and only if ____________________________________ converges.

1e. Comparison Test for a positive-termed series $\sum a_n$ where $a_n \geq 0$.

- If $0 \leq a_n \leq b_n$ for all $n \in \mathbb{N}$ and $\sum b_n$ __________________, then $\sum a_n$ __________________.
- If $0 \leq b_n \leq a_n$ for all $n \in \mathbb{N}$ and $\sum b_n$ __________________, then $\sum a_n$ __________________.

1f. Limit Comparison Test for a positive-termed series $\sum a_n$ where $a_n \geq 0$.
Let $b_n > 0$ and $\lim_{n\to\infty} \frac{a_n}{b_n} = L$.
If _________ $< L <$ _________, then $\sum a_n$ converges if and only if __________________ .

1g. Ratio and Root Tests for a positive-termed series $\sum a_n$ where $a_n \geq 0$.
Let $\rho = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$ or $\rho = \lim_{n\to\infty} (a_n)^{\frac{1}{n}}$.

- If ρ _________ then $\sum a_n$ converges.
- If ρ _________ then $\sum a_n$ diverges.
- If ρ _________ then the test is inconclusive.

1h. Alternating Series Test for an alternating series $\sum (-1)^n a_n$ where $a_n > 0$ for each $n \in \mathbb{N}$.
If

- a_n _________ a_{n+1} for each $n \in \mathbb{N}$
- $\lim_{n\to\infty} a_n =$ _________

then $\sum (-1)^n a_n$ ___________________________

1i. By definition, for an arbitrary series $\sum a_n$, (fill in the blanks with converges or diverges).

- $\sum a_n$ is absolutely convergent if and only if $\sum |a_n|$ ____________________
- $\sum a_n$ is conditionally convergent if and only if $\sum a_n$ __________ and $\sum |a_n|$ __________
- $\sum a_n$ is divergent if and only if $\sum a_n$ ____________________

1j. Fill in the 3 blank lines (with absolutely convergent, conditional convergent, or divergent) on the following FLOW CHART for class used to determine if a series $\sum_{n=17}^{\infty} a_n$ is: absolutely convergent, conditional convergent, or divergent.

Does $\sum |a_n|$ converge?
Since $|a_n| \geq 0$, use a positive term test: integral test, CT, LCT, ratio/root test.

if NO $\Rightarrow$ Does $\lim_{n\to\infty} |a_n| = 0$?

if NO $\Rightarrow$ $\sum a_n$ is ________________

if YES $\Downarrow$ (from "Does $\sum |a_n|$ converge?")

$\sum a_n$ is ______________________________

if YES $\Downarrow$ (from "Does $\lim_{n\to\infty} |a_n| = 0$?")

Is $\sum a_n$ an alternating series?

if YES $\Downarrow$

Does $\sum a_n$ satisfy the conditions of the Alternating Series Test?

if YES $\Downarrow$

$\sum a_n$ is ______________________________

2. Circle T if the statement is TRUE. Circle F if the statement if FALSE.

T F If $\lim_{n\to\infty} a_n = 0$, then $\sum a_n$ converges

T F If $\sum a_n$ converges, then $\lim_{n\to\infty} a_n = 0$.

T F If $\sum a_n$ converges and $\sum b_n$ converge, then $\sum (a_n + b_n)$ converges.

T F If $\sum (a_n + b_n)$ converges, then $\sum a_n$ converges and $\sum b_n$ converge.

T F If $S_N = \sum_{n=1}^{N} r^n$, then $S_N = \dfrac{r - r^{N+1}}{1-r}$.

3. Check the correct box and then indicate your reasoning below. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

$$\sum_{n=17}^{\infty} \frac{(-1)^n}{n}$$

- ☐ absolutely convergent
- ☐ conditionally convergent
- ☐ divergent

4. Check the correct box and then indicate your reasoning below. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n\ (n+1)\ (n+2)}}$$

☐ absolutely convergent

☐ conditionally convergent

☐ divergent

5. Let

$$a_n = \frac{n!}{(2n-1)!}$$

5a. Find an expression for $\frac{a_{n+1}}{a_n}$ that does NOT have a fractorial sign (that is a ! sign) in it.

$$\frac{a_{n+1}}{a_n} =$$

5b. Check the correct box and then indicate your reasoning below. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

$$\sum_{n=1}^{\infty} (-1)^n \frac{n!}{(2n-1)!}$$

☐ absolutely convergent

☐ conditionally convergent

☐ divergent

6. Consider the formal power series

$$\sum_{n=1}^{\infty} \frac{(5x+10)^n}{n}$$

Hint: $(5x+10)^n \;=\; [5\,(x+2)]^n \;=\; 5^n\,(x+2)^n \;=\; 5^n\,(x-(-2)\,)^n$

The center is $x_0 =$ ________________ and the radius of convergence is $R =$ ________________ .

As we did in class, make a number line indicating where the power series is: absolutely convergent, conditionally convergent, and divergent. Indicate your reasoning and specifically specify what test(s) you are using. Don't forget to check the endpoints, if there are any.

$$\longleftrightarrow$$

7. Consider the formal power series

$$\sum_{n=2}^{\infty} \frac{x^n}{(\ln n)^n} \ .$$

Hint 1: $\frac{x^n}{(\ln n)^n} \ = \ \left[\frac{x}{\ln n}\right]^n$ so would you rather use the root test or the ratio test?

Hint 2: $\ln(a^r) \ = \ r\ln(a)$ but $(\ln(a))^r \ \neq \ r\ln(a) \ +$

The center is $x_0 =$ __________________ and the radius of convergence is $R =$ __________________ .
As we did in class, make a number line indicating where the power series is: absolutely convergent, conditionally convergent, and divergent. Indicate your reasoning and specifically specify what test(s) you are using. Don't forget to check the endpoints, if there are any.

8. Fill-in the 6 blanks.

Consider the power series

$$\sum_{n=1}^{\infty} (-1)^n \; a_n x^n$$

where all of the a_n's are positive. Let's say that you know that

if	$0 < x < 17$	then $\sum (-1)^n \; a_n x^n$ converges
if	$x = 17$	then $\sum (-1)^n \; a_n x^n$ conditionally converges
if	$17 < x$	then $\sum (-1)^n \; a_n x^n$ diverges .

Then this power series has:

center at $x_0 =$ ____________________ and radius of convergence $R =$ ____________________ .

Also, what can you say about the following interval? Fill in the blanks below with:

- is absolutely convergent
- is conditionally convergent
- is divergent
- inconclusive (not enough information given to decide in general).

if	$-17 < x < 0$	then $\sum (-1)^n \; a_n x^n$	______________________________
if	$x < -17$	then $\sum (-1)^n \; a_n x^n$	______________________________
if	$x = 0$	then $\sum (-1)^n \; a_n x^n$	______________________________
if	$x = -17$	then $\sum (-1)^n \; a_n x^n$	______________________________