Warning: there are 5 problems: 2 problems on the front and 3 problems on the back. Turn your paper over

NAME: ____________________

There are 5 problems.
Each problem is worth 2 points.

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please check the box of your section

☐ **Section 005 (WF 8:00 am)**

or

☐ **Section 006 (WF 9:05 am)**

INSTRUCTIONS: Indicate your reasoning. Put answers in box and show work below the box.

1. $\boxed{\lim_{n\to\infty} \frac{1}{2^n} = 0}$

$$\lim_{n\to\infty} \frac{1}{2^n} = \lim_{n\to\infty} \left(\frac{1}{2}\right)^n = 0 \quad \leftarrow r = \tfrac{1}{2},\ |r|<1$$

or

$$\lim_{n\to\infty} \frac{1}{2^n} = \cancel{\frac{1}{\infty}} = 0.$$

2. $\boxed{\lim_{n\to\infty} (17)^n = \infty \text{ or diverges or DNE}}$

$$\lim_{n\to\infty} (17)^n = \infty, \quad r = 17, \quad |r|>1.$$

3. $\boxed{\lim_{n\to\infty} \frac{5n^3+6n+3}{17n^3+9n^2+4} = \frac{5}{17}}$

$$\lim_{n\to\infty} \frac{5n^3+6n+3}{17n^3+9n^2+4} = \lim_{n\to\infty} \frac{5+\frac{6}{n^2}+\frac{3}{n^3}}{17+\frac{9}{n}+\frac{4}{n^3}} = \frac{5+0+0}{17+0+0}$$

divide numerator & denom. by n (highest power you see) $= n^3$

4. $\boxed{\lim_{n\to\infty} \frac{n^3}{e^n} \underset{L'H}{\overset{\frac{\infty}{\infty}}{=}} \lim_{n\to\infty} \frac{3n^2}{e^n} \underset{L'H}{\overset{\frac{\infty}{\infty}}{=}} \lim_{n\to\infty} \frac{6n}{e^n} \underset{L'H}{\overset{\frac{\infty}{\infty}}{=}} \lim_{n\to\infty} \frac{6}{e^n} = \cancel{\frac{1}{\infty}} = 0}$

did up here

Helpful intuition: $y=e^x$ grows so much faster than $y=x^3$ that the answer should be zero. We know, for $0<p<\infty$ and n big enough: $n^p < e^n$. So how to show the limit is zero. WAY #1 is to use L'Hôpital's rule. WAY #2 is to use the Squeeze Theorem.

$$0 \le \frac{n^3}{e^n} \le \frac{n^3}{n^4} = \frac{1}{n} \xrightarrow{n\to\infty} 0$$

5. $\boxed{\lim_{n\to\infty} \frac{\sqrt[3]{n^2+5}}{\sqrt[6]{64n^4+17n}} = \left(\frac{1}{64}\right)^{1/6} \text{ or } \frac{1}{2}}$

hint:

$$\frac{\sqrt[3]{n^2+5}}{\sqrt[6]{64n^4+17n}} = \frac{(n^2+5)^{\frac{1}{3}}}{(64n^4+17n)^{\frac{1}{6}}} = \frac{(n^2+5)^{\frac{2}{6}}}{(64n^4+17n)^{\frac{1}{6}}} = \frac{[(n^2+5)^2]^{\frac{1}{6}}}{(64n^4+17n)^{\frac{1}{6}}} =$$

$$= \left[\frac{n^4+10n^2+25}{64n^4+17n}\right]^{1/6} = \left[\frac{1+\frac{10}{n^2}+\frac{25}{n^4}}{64+\frac{17}{n^3}}\right]^{1/6} \xrightarrow{n\to\infty} \left[\frac{1+0+0}{64+0}\right]^{1/6}$$

$\boxed{\text{÷ num. \& dem. by } n^4 \text{ (see above problem 3)}}$

using: $\lim_{n\to\infty}(a_n^p) = \left(\lim_{n\to\infty} a_n\right)^p$

$\boxed{\text{NO!}}$

Remark: Unfortunately I saw on too many papers:

$(n^2+5)^{\frac{1}{3}} = n^{\frac{2}{3}} + 5^{\frac{1}{3}}$. Eg: $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$.