| MARK BOX | | |
|---|---|---|
| PROBLEM | POINTS | |
| 1 a–y | 25 | |
| 2 | 10 | |
| 3 | 11 | |
| 4 | 11 | |
| 5 | 11 | |
| 6 | 11 | |
| 7 | 11 | |
| 8 take home | 10 | |
| 9 extra credit take home | 2 | |
| % | 100 | |

**NAME:** ______________________________

**please check the box of your section**

☐ **Section 005 (WF 8:00 am)**

or

☐ **Section 006 (WF 9:05 am)**

**INSTRUCTIONS**:

(1) To receive credit you must:
  (a) **work in a logical fashion, show all your work, indicate your reasoning**; **no credit will be given for an answer that *just appears***; such explanations help with partial credit
  (b) if a line/box is provided, then:
  — show you work BELOW the line/box
  — put your answer on/in the line/box
  (c) if no such line/box is provided, then box your answer
(2) The MARK BOX indicates the problems along with their points. Check that your copy of the exam has all of the problems.
(3) You may **not** use a calculator, books, personal notes.
(4) During this exam, do not leave your seat. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.
(5) This exam covers (from *Calculus* by Anton, Bivens, Davis $8^{\text{th}}$ ed.): Sections 8.2, 8.3, 8.4, 8.5, 8.7, 8.8. .

**Problem Inspiration**:

**1.** You were warned.
**2.** An example from class.
**3.** Handout of 100 integrals # 7
**4.** Handout of 100 integrals # 23
**5.** Handout of 100 integrals # 47
**6.** Handout of 100 integrals # 26
**7.** Handout of 100 integrals - 4th page
**8.** just like the homework.
**9.** Section 8.8, number 16.

**Hints:**

(1) **You can check your answers to the indefinite integrals by differentiating.**
(2) **For more partial credit, box your $u - du$ substitutions.**

**1. Fill in the blanks (each worth 1 point).**

**1a.** $\int \frac{du}{u} \;=\;$ __________ $|u| + C$

**1b.** If $a$ is a constant and $a > 0$ but $a \neq 1$, then $\int a^u \, du \;=\;$ ______________________________ $+C$

**1c.** $\int \cos u \, du \;=\;$ ____________________________________________________________ $+C$

**1d.** $\int \sec^2 u \, du \;=\;$ ____________________________________________________________ $+C$

**1e.** $\int \sec u \tan u \, du \;=\;$ ____________________________________________________________ $+C$

**1f.** $\int \sin u \, du \;=\;$ ____________________________________________________________ $+C$

**1g.** $\int \csc^2 u \, du \;=\;$ ____________________________________________________________ $+C$

**1h.** $\int \csc u \cot u \, du \;=\;$ ____________________________________________________________ $+C$

**1i.** $\int \tan u \, du \;=\;$ ____________________________________________________________ $+C$

**1j.** $\int \cot u \, du \;=\;$ ____________________________________________________________ $+C$

**1k.** $\int \sec u \, du \;=\;$ ____________________________________________________________ $+C$

**1l.** $\int \csc u \, du \;=\;$ ____________________________________________________________ $+C$

**1m.** If $a$ is a contant and $a > 0$ then $\int \frac{1}{\sqrt{a^2-u^2}} \, du \;=\;$ ________________________________________ $+C$

**1n.** If $a$ is a contant and $a > 0$ then $\int \frac{1}{a^2+u^2} \, du \;=\;$ ________________________________________ $+C$

**1o.** If $a$ is a contant and $a > 0$ then $\int \frac{1}{u\sqrt{u^2-a^2}} \, du \;=\;$ ____________________________________ $+C$

**1p.** Partial Fraction Decomposition. If one wants to integrate $\frac{f(x)}{g(x)}$ where $f$ and $g$ are polyonomials and [degree of $f$] $\geq$ [degree of $g$], then one must first do ______________________________

**1q.** Integration by parts formula: $\int u \, dv \;=\;$ ____________________________________________

**1r.** Trig substitution: (recall that the *integrand* is the function you are integrating)
if the integrand involves $a^2 - u^2$, then one makes the substitution $u \;=\;$ __________________________

**1s.** Trig substitution:
if the integrand involves $a^2 + u^2$, then one makes the substitution $u \;=\;$ __________________________

**1t.** Trig substitution:
if the integrand involves $u^2 - a^2$, then one makes the substitution $u \;=\;$ __________________________

**1u.** trig formula ... your answer should involve trig functions of $\theta$, and not of $2\theta$: $\sin(2\theta) =$ ________________ .

**1v.** trig formula ... $\cos(2\theta)$ should appear in the numerator: $\cos^2(\theta) \;=\; \frac{\rule{6cm}{0.4pt}}{2}$ .

**1w.** trig formula ... $\cos(2\theta)$ should appear in the numerator: $\sin^2(\theta) \;=\; \frac{\rule{6cm}{0.4pt}}{2}$ .

**1x.** trig formula ... since $\cos^2\theta + \sin^2\theta = 1$, we know that the corresponding relationship beween tangent (i.e., tan) and secant (i.e., sec) is ____________________________________________________ .

**1y.** $\arcsin(\,\mathbf{-}\frac{\sqrt{2}}{2}) \;=\;$ ________________ **RADIANS**. (your answer should be an angle)

**2.**

$$\int_{x=17}^{x=\infty} \frac{1}{x^2}\ dx \ =$$

**3.**

**3a.** $D_x \ \tan x =$

**3b.** $D_x \ \sec x =$

**3c.** $\int \tan^2 x \ dx \ =$ $+ \text{C}$

Hint: problems 1x, 3a, and 3b might come in handy for problem 3c.

**4.**

$$\int \ln(3x+6)\ dx \ = \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad + \text{C}$$

Hint: be clever in your choice of $v$ to avoid unneeded work.

Note that $\frac{3}{3x+6} = \frac{3}{3(x+2)} = \frac{1}{x+2}$

**5.**

$$\int \frac{x^4 + 2x + 2}{x^5 + x^4}\, dx \ = \qquad\qquad\qquad + \text{C}$$

Hint: $x^5 + x^4 = (x^4)(x+1) = (x-0)^4\ (x+1)^1$ .

**6.** $$\int \frac{x^2}{\sqrt{16 - x^2}}\ dx \ = \qquad\qquad\qquad\qquad + \text{C}$$

Hint: problems 1 u - w might come in handy.

**7.**

$$\int \csc^3 x \; dx \;=\; \qquad\qquad\qquad\qquad\qquad\qquad + \text{C}$$

Hint: bring to the other side idea, similar to how to do $\int \sec^3 x dx$. Helpful:

- $D_x \cot x = -\csc^2 x$
- $D_x \csc x = -\csc x \cot x$
- $\cot^2 x + 1 = \csc^2 x$.

**8.** Numerical Integration . Let

$$I = \int_{x=1}^{x=3} \frac{1}{x^2} \, dx \; .$$

The 10 steps of this problem are similar to the homework but the number of subintervals is **6** and **not 10**. **On the parts that say "Do not use a calculator", you need not do alot of arthritic.**

**8-1.** Find the exact value of $I$, without using a calculator. Your answer should be a fraction, without decimal places.

| $I =$ |
|---|

**8-2.** Find an approximation for $I$, using part 1 and your calculator, to as many decimal places as your calculator will give you.

| $I \approx$ |
|---|

**8-3.** Approximate $I$ using the Trapezoid Rule $T_n$ with $n = 6$ subintervals. Do not use a calculator (so your answer will have several numbers added together).

| $T_6 =$ |
|---|

**8-4.** Find an approximation for $T_6$, using part 3 and your calculator, to as many decimal places as your calculator will give you.

| $T_6 \approx$ |
|---|

**8-5.** Approximate $I$ using Simpon's Rule $S_{2n}$ with $2n = 6$ subintervals. Do not use a calculator (so your answer will have several numbers added together).

$S_6 =$

**8-6.** Find an approximation for $S_6$, using part 5 and your calculator, to as many decimal places as your calculator will give you.

$S_6 \approx$

**8-*.** Find the first 4 derivatives of $f(x) = x^{-2}$.

**8-7.** Use inequality (11), page 563, to find an upper bound on the error in part 3. Do not use a calculator.

| |
|---|
| $\lvert T_6 - I \rvert \leq$ |

**8-8.** Use your calculator to approximate the error estimate in part 7 to as many decimal places as your calculator will give you.

| |
|---|
| $\lvert T_6 - I \rvert \lessapprox$ |

**8-9.** Use inequality (12), page 563 to find an upper bound on the error in part 5. Do not use a calculator.

| |
|---|
| $\lvert S_6 - I \rvert \leq$ |

**8-10.** Use your calculator to approximate the error iestimate in part 9, to as many decimal places as your calculator will give you.

| |
|---|
| $\lvert S_6 - I \rvert \lessapprox$ |

**9.** Extra Credit - number 16 from section 8.8.

$$\int_{x=-\infty}^{x=\infty} \frac{e^{-x}}{1+e^{-2x}}\,dx =$$