

MARK BOX		
PROBLEM	POINTS	
1 a-j	10	
2	10	
3a Extra Credit	(1)	
3b	10	
3c	10	
3d	10	
4	10	
5	10	
6	10	
7	10	
8	10	
%	100	

NAME (printed): Key

SIGNATURE: _____

please check the box of your section below

 Section 005 (W&F 8:00 am)

or

 Section 006 (W&F 9:05 am)**INSTRUCTIONS:**

- (1) To receive credit you must:
 - (a) work in a logical fashion, show all your work, indicate your reasoning;
no credit will be given for an answer that just appears;
such explanations help with partial credit
 - (b) when applicable put your answer on/in the line/box provided
 - (c) if no such line/box is provided, then box your answer
- (2) The MARK BOX indicates the problems along with their points.
Check that your copy of the exam has all of the problems.
- (3) You may **not** use a calculator, books, personal notes.
- (4) During this exam, do not leave your seat. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.
- (5) This exam covers (from *Calculus* by Anton, Bivens, Davis 8th ed.):
Section 7.1 – 7.4, 7.6, 7.7, 8.1 .

Problem Inspiration:

1. Math 141 HANDOUT, Spring 2006 Exam 1 # 1
2. homework problems from § 7.1, , actually problem § 7.1 # 34
3. homework problems from § 7.2 and 7.3 , homework problem § Ch. 7 Review # 6,
Spring 2006 Exam 1 # 3, and examples from the lectures over § 7.2 & § 7.3
4. homework problem § 7.4 # 9 & 11 and Spring 2006 Exam 1 # 4
5. homework problem § 8.1 # 6
6. homework problem § 8.1 # 6
7. homework problem § 8.1 # 11
8. homework problem § 8.1 # 15

If you missed \pm
 $1c$, $1e$, or $1f$,
 then see the quiz posted on
 Ch8 homework page for §8.1

1. Fill in the blanks (each worth 1 point).

1a. $\int \frac{du}{u} = \ln |u| + C$

1b. $\int e^u du = e^u + C$

1c. If a is a constant and $a > 0$ but $a \neq 1$, then

$\int a^u du = \frac{a^u}{\ln a} + C$

1d. $\int \cos u du = \sin u + C$

1e. $\int \tan u du = -\ln |\cos u| + C \cong \ln |\sec u| + C$

1f. $\int \sec u du = \ln |\sec u + \tan u| + C \cong -\ln |\sec u - \tan u| + C$

1g. $\int \sec^2 u du = \tan u + C$

1h. If a is a constant and $a > 0$ then

$\int \frac{1}{\sqrt{a^2 - u^2}} du = \sin^{-1} \frac{u}{a} + C$

1i. If a is a constant and $a > 0$ then

$\int \frac{1}{a^2 + u^2} du = \frac{1}{a} \tan^{-1} \frac{u}{a} + C$

1j. The integral of $y = f(x)$ with respect to x is denoted by $\int f(x) dx$.

The integral of $x = g(y)$ with respect to y is denoted by $\int g(y) dy$.

2. Let R be the region between the curve

$$y = \sin x$$

and the line segment joining the points

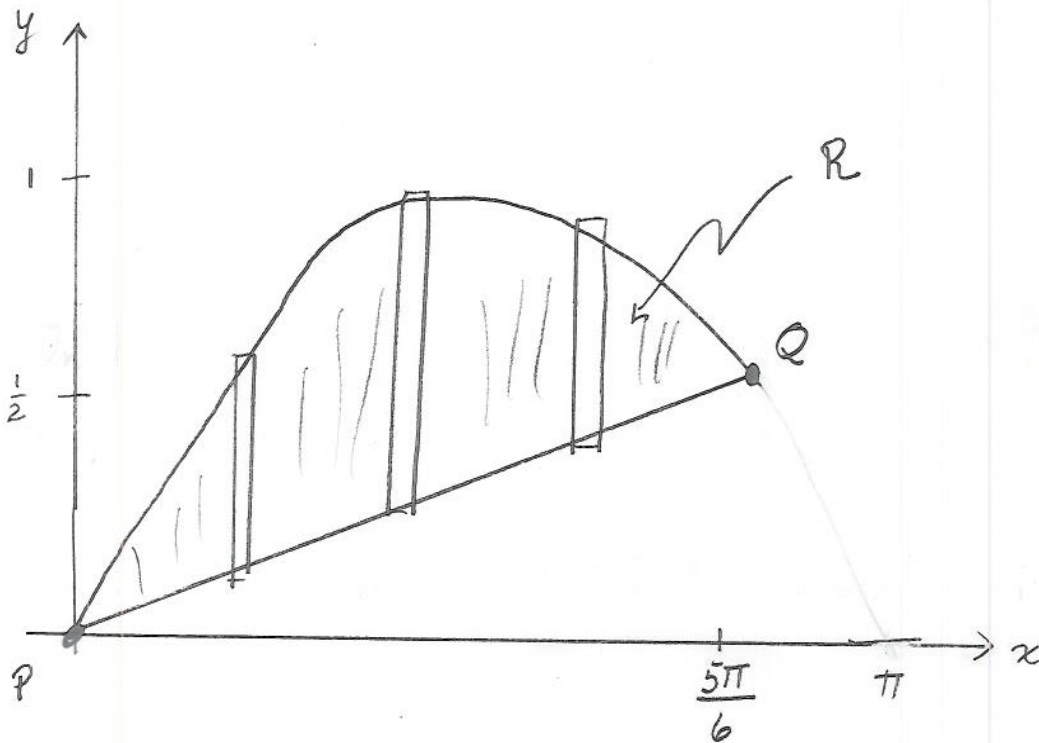
$$P = (0, 0) \quad \text{and} \quad Q = \left(\frac{5\pi}{6}, \frac{1}{2} \right).$$

Let A be the area of the region R .

2a. Make a rough sketch of the region R , labeling P and Q .

The equation of the line through points P and Q is: $y = \frac{3x}{5\pi}$.

$$\text{slope of line thru } P \& Q = \frac{\Delta y}{\Delta x} = \frac{\frac{1}{2} - 0}{\frac{5\pi}{6} - 0} = \frac{1}{2} \cdot \frac{6}{5\pi} = \frac{3}{5\pi}$$



2b. Express the area A as **ONE** integral (but not 2 or more integrals).

You do NOT have to evaluate the integral(s) nor do lots of algebra.

$$A = \int_{x=0}^{x=\frac{5\pi}{6}} \left(\sin x - \frac{3x}{5\pi} \right) dx$$

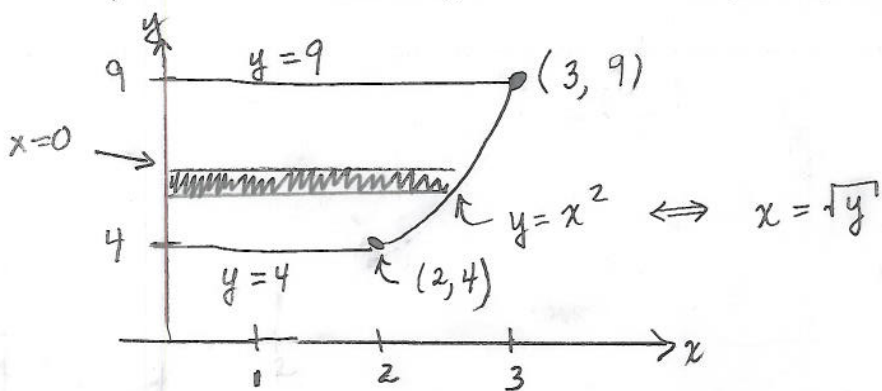
(3.a) Sketched below is the region R that is enclosed by

$$x = 0 \quad \text{and} \quad y = 4 \quad \text{and} \quad y = 9 \quad \text{and} \quad y = x^2.$$

In each of problems 3a, 3b, 3c; 3d,

- R will be revolved around some line to create a solid of revolution
- using either the disk, washer, or shell method, express the volume V of the resulting solid of revolution as one integral (and NOT as 2 or more integrals).
- In the space provided **below** each problem, make some *good enough sketch* (does not have to be too fancy) to indicate (i.e., help justify) your thinking/reasoning behind your solution
- you do not have to do lots of algebra to your integrand
- you do not have to integrate your integral.

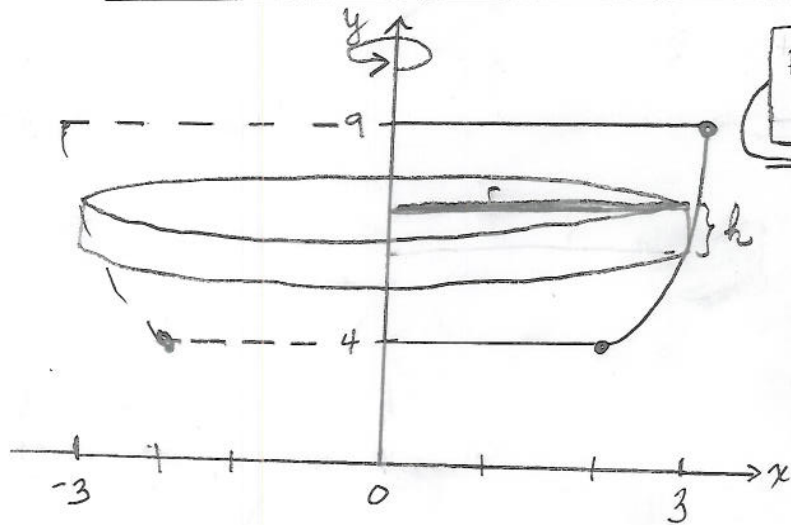
Extra Credit/Hint In the sketch below, draw in a typical rectangle (should it be horizontal or vertical?) that would be used to express the area of R as precisely 1 integral (and not 2 integrals).



3b

The volume V of the solid obtained by revolving the region R about the y -axis is

$$V = \int_{y=4}^{y=9} \pi (\sqrt{y})^2 dy \quad \text{or} \quad \pi \int_{y=4}^{y=9} y dy$$

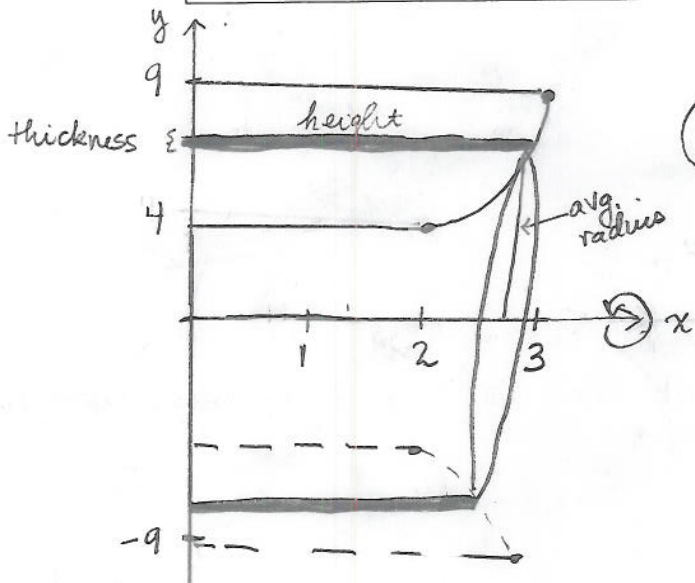


[partition y -axis (to get 1 integral)
 revolving about y -axis
 no hole
 Disk Method / all in terms of y

Volume of typical element
 = volume of a disk
 = (area base) (height)
 = $(\pi r^2) h$
 = $\pi (\sqrt{y})^2 \Delta y$

3c. The volume V of the solid obtained by revolving the region R about the x -axis is

$$V = \int_{y=4}^{y=9} 2\pi y \cdot \sqrt{y} \, dy \quad \text{or} \quad 2\pi \int_{y=4}^{y=9} y^{3/2} \, dy$$



partition y -axis (to get 1 integral)
& revolving about x -axis

⇒ Shell Method / all in terms of y

Volume of typical element

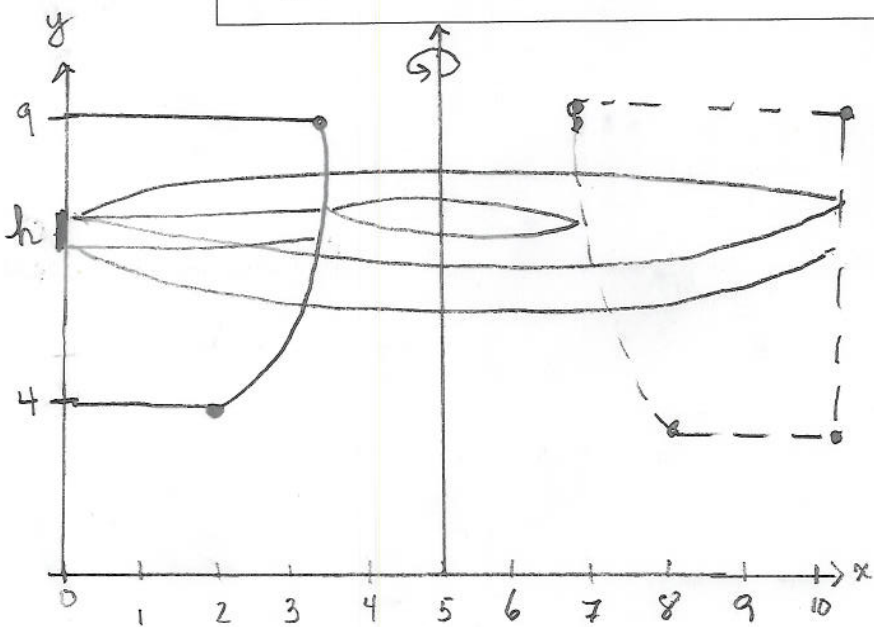
= volume of a shell

= 2π (avg. radius) (height) (thickness)

= $2\pi y \sqrt{y} \, \Delta y$

3d. The volume V of the solid obtained by revolving the region R about the vertical line $x = 5$ is

$$V = \int_{y=4}^{y=9} \pi (5^2 - (5 - \sqrt{y})^2) \, dy \quad \text{or} \quad \pi \int_{y=4}^{y=9} (10y^{1/2} - y) \, dy$$



partition y -axis (to get 1 integral)
revolve around a line parallel to y -axis
have a hole

⇒ Washer method / all in terms of y

Volume of typical element

= volume of a washer

= Volume_{big} - Volume_{little}

= $\pi r_{\text{big}}^2 h - \pi r_{\text{little}}^2 h$

= $\pi (r_{\text{big}}^2 - r_{\text{little}}^2) h$

= $\pi (5^2 - (5 - \sqrt{y})^2) \Delta y$

4. Express the arclength of the parameterized curve

$$x(t) = t^2 + 4$$

$$y(t) = t + 5$$

from the point

$$P = (4, 5)$$

to the point

$$Q = (13, 8)$$

as an integral with respect to t .

$$\text{arclength} = \int_{t=0}^{t=3} \sqrt{(2t)^2 + 1^2} dt \equiv \int_{t=0}^{t=3} \sqrt{4t^2 + 1} dt$$

Once again, you do not have to do lots of algebra to your integrand nor integrate your integral.

$$P = (4, 5) \Leftrightarrow \begin{cases} 4 = t^2 + 4 \\ 5 = t + 5 \end{cases} \Leftrightarrow t = 0$$

$$Q = (13, 8) \Leftrightarrow \begin{cases} 13 = t^2 + 4 \\ 8 = t + 5 \end{cases} \Leftrightarrow t = 3$$

$$\frac{dx}{dt} = \frac{d}{dt} (t^2 + 4) = 2t$$

$$\frac{dy}{dt} = \frac{d}{dt} (t + 5) = 1$$

$$\text{arclength} = \int_{t=a}^{t=b} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

5.

$$\int \frac{1}{9+25x^2} dx = \frac{1}{15} \tan^{-1} \left(\frac{5x}{3} \right) + C \quad \text{or} \quad \frac{1}{15} \arctan \left(\frac{5x}{3} \right) + C$$

Remark: box your substitution box for more partial credit.

Recall: you can check your answer via differentiation (if you have time).

$$\text{(WAY \#1)} \quad \int \frac{dx}{9+25x^2} = \int \frac{dx}{3^2+(5x)^2} = \frac{1}{5} \int \frac{5dx}{3^2+(5x)^2} = \frac{1}{5} \int \frac{du}{3^2+u^2}$$

$$\begin{array}{l} u = 5x \\ du = 5dx \end{array}$$

$$= \frac{1}{5} \cdot \frac{1}{3} \tan^{-1} \frac{u}{3} + C = \frac{1}{15} \tan^{-1} \frac{5x}{3} + C$$

↑
a=3

See problem 1!

(WAY #2)

$$\int \frac{dx}{9+25x^2} = \frac{1}{9} \int \frac{dx}{1+\frac{25x^2}{9}} = \frac{1}{9} \int \frac{dx}{1+(\frac{5x}{3})^2}$$

want 1 here so

$$\begin{array}{l} u = \frac{5}{3}x \\ du = \frac{5}{3}dx \end{array}$$

$$= \frac{1}{9} \cdot \frac{3}{5} \int \frac{\frac{5}{3} dx}{1+(\frac{5x}{3})^2}$$

$$= \frac{1}{3 \cdot 5} \int \frac{du}{1+u^2} = \frac{1}{15} \tan^{-1} u + C$$

$$= \frac{1}{15} \tan^{-1} \frac{5x}{3} + C$$

Check
ans.

$$D_x \tan^{-1} u = \frac{1}{1+u^2} \frac{du}{dx}$$

$$D_x \left(\frac{1}{15} \tan^{-1} \frac{5x}{3} \right) = \frac{1}{15} \cdot \frac{1}{1+(\frac{5x}{3})^2} \cdot \frac{5}{3} = \frac{1}{15} \cdot \frac{5}{3} \cdot \frac{1}{1+\frac{25x^2}{9}}$$

$$= \frac{1}{3 \cdot 3} \cdot \frac{1}{\frac{9+25x^2}{9}} = \frac{1}{3 \cdot 3} \cdot \frac{9}{9+25x^2} = \frac{1}{9+25x^2} \quad \text{☺}$$

6.

$$\int \frac{x}{9+25x^2} dx = \frac{1}{50} \ln |9+25x^2| + C \cong \frac{1}{50} \ln (9+25x^2) + C$$

Remark: box your substitution box for more partial credit.

Recall: you can check your answer via differentiation (if you have time).

$$\begin{aligned} u &= 9+25x^2 \\ du &= 50x dx \end{aligned}$$

\bullet $9+25x^2$ is always positive so $|9+25x^2| = 9+25x^2$

$$\int \frac{x}{9+25x^2} dx = \frac{1}{50} \int \frac{50x dx}{9+25x^2} = \frac{1}{50} \int \frac{du}{u} = \frac{1}{50} \ln |u| + C = \frac{1}{50} \ln |9+25x^2| + C$$

Check
Answer

$$D_x \frac{1}{50} \ln |9+25x^2| = \frac{1}{50} \cdot \frac{1}{9+25x^2} \cdot 50x = \frac{x}{9+25x^2} \quad \text{☺}$$

7.

$$\int \cos^{17}(7x) \sin(7x) dx = \frac{-1}{7 \cdot 18} \cos^{18}(7x) + C \quad \text{or} \quad \frac{-1}{126} \cos^{18}(7x) + C$$

Remark: box your substitution box for more partial credit.

Recall: you can check your answer via differentiation (if you have time).

$$u = \cos(7x)$$

$$du = -7 \sin(7x) dx$$

$$\begin{aligned} \int \cos^{17}(7x) \sin(7x) dx &= -\frac{1}{7} \int [\cos(7x)]^{17} (-7 \sin(7x) dx) \\ &= -\frac{1}{7} \int u^{17} du = -\frac{1}{7} \frac{u^{18}}{18} + C = -\frac{1}{7 \cdot 18} [\cos(7x)]^{18} \end{aligned}$$

Check
Answer

$$D_x \left[-\frac{1}{7 \cdot 18} \cos^{18}(7x) \right] = -\frac{1}{7 \cdot 18} \cdot 18 \cos^{17}(7x) \cdot D_x \cos(7x)$$

$$= -\frac{1}{7 \cdot 18} \cdot 18 \cos^{17}(7x) (-\sin(7x)) (7) = \cos^{17}(7x) \sin(7x) \quad \text{😊}$$

8

$$\int \frac{e^{\sqrt{3x-1}}}{\sqrt{3x-1}} dx = \frac{2}{3} e^{\sqrt{3x-1}} + C$$

Remark: box your substitution box for more partial credit.

Recall: you can check your answer via differentiation (if you have time).

$$\int \frac{e^{\sqrt{3x-1}}}{\sqrt{3x-1}} dx = \int (3x-1)^{-1/2} e^{(3x-1)^{1/2}} dx$$

WAY #1

$$u = (3x-1)^{1/2}$$

$$du = \frac{3}{2} (3x-1)^{-1/2} dx$$

$$\int \frac{e^{\sqrt{3x-1}}}{\sqrt{3x-1}} dx = \frac{2}{3} \int e^{(3x-1)^{1/2}} \left(\frac{3}{2} (3x-1)^{-1/2} dx \right) = \frac{2}{3} \int e^u du$$

$$= \frac{2}{3} e^u + C = \frac{2}{3} e^{\sqrt{3x-1}} + C$$

WAY #2

$$u = e^{(3x-1)^{1/2}}$$

$$du = e^{(3x-1)^{1/2}} \cdot \frac{3}{2} (3x-1)^{-1/2} dx$$

$$\int \frac{e^{\sqrt{3x-1}}}{\sqrt{3x-1}} dx = \frac{2}{3} \int e^{(3x-1)^{1/2}} \left(\frac{3}{2} (3x-1)^{-1/2} dx \right) = \frac{2}{3} \int du$$

$$= \frac{2}{3} u + C = \frac{2}{3} e^{(3x-1)^{1/2}} + C$$

Check Answer

$$D_x \frac{2}{3} e^{\sqrt{3x-1}} = \frac{2}{3} D_x e^{(3x-1)^{1/2}}$$

$$= \frac{2}{3} e^{(3x-1)^{1/2}} \cdot D_x (3x-1)^{1/2} = \frac{2}{3} e^{(3x-1)^{1/2}} \cdot \frac{1}{2} (3x-1)^{-1/2} (3)$$

$$= e^{(3x-1)^{1/2}} (3x-1)^{-1/2} = \frac{e^{\sqrt{3x-1}}}{\sqrt{3x-1}}$$

