Prof. Girardi	Math 142.003/004	Fall 2005	12.08.05	Final Exam

MARK BOX		
PROBLEM	POINTS	
1 a – r	35	
2	5	
3	5	
4	5	
5	5	
6	5	
7	5	
8	5	
9	5	
10	5	
11	5	
12	5	
13	5	
14	5	
%	100	

NAME: ______________________________

SSN: ______________________________

please check the box of your section below

☐ **Section 003 (MW 9:05 pm)**

or

☐ **Section 004 (MW 10:10 pm)**

INSTRUCTIONS:

(1) To receive credit you must:
   (a) **work in a logical fashion, show all your work, indicate your reasoning**; **no credit will be given for an answer that *just appears***; such explanations help with partial credit
   (b) when applicable put your answer on/in the line/box provided
   (c) if no such line/box is provided, then box your answer
(2) The MARK BOX indicates the problems along with their points. Check that your copy of the exam has all of the problems.
(3) You may **not** use a calculator, books, personal notes.
(4) During this exam, do not leave your seat. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.
(5) This exam covers (from *Calculus* by Anton, Bivens, Davis 8^{th} ed.): Sections 6.6, 6.8, 6.9, 7.1–7.4, 7.6, 7.7, 8.1–8.8, 10.1–10.10. .

Problem Inspiration: homework and old exams.

Solutions will be available on the course homepage later this afternoon.

1. Fill in the blanks (each worth 1 point) and boxes (each worth 2 points).

1a. $\int \frac{dx}{x} =$ ________ $|x| + C$

1b. $D_x\ e^x =$ ________________________

1c. If $a > 0$ but $a \neq 1$, then $D_x\ a^x =$ ________________________
Hint: $a^x = e^{\ln(a^x)} = e^{x \ln(a)}$. Your answer should **not** have an "e" in it.

1d. $D_x\ \tan x =$ ________________________

1e. $\int \sec x\ \tan x =$ ________________________________ $+ C$

1f. Integration by parts formula:
$\int u\, dv =$ ____

1g. Trig substitution: (recall that the *integrand* is the function you are integrating)
if the integrand involves $a^2 - u^2$, then one makes the substitution $u =$ ________________________

1h. Trig substitution:
if the integrand involves $a^2 + u^2$, then one makes the substitution $u =$ ________________________

1i. Partial Fraction Decomposition. If one wants to integrate $\frac{f(x)}{g(x)}$ where f and g are polyonomials

and [degree of f] $\geq$ [degree of g], then one must first do ____________________________

1j. Integral Test: $a_n > 0$
Let $f\colon [1, \infty) \to \mathbb{R}$ be so that

- $a_n = f($________$)$ for each $n \in \mathbb{N}$
- f is a ________________________________ function
- f is a ________________________________ function
- f is a ________________________________ function .

Then $\sum a_n$ converges if and only if ________________________________ converges.

1k. Comparison Test: $a_n > 0$

- If $0 \leq a_n \leq b_n$ for all $n \in \mathbb{N}$ and $\sum b_n$ ________________, then $\sum a_n$ ________________.
- If $0 \leq b_n \leq a_n$ for all $n \in \mathbb{N}$ and $\sum b_n$ ________________, then $\sum a_n$ ________________.

1l. Limit Comparison Test: $a_n > 0$
Let $b_n > 0$ and $\lim_{n \to \infty} \frac{a_n}{b_n} = L$.
If ________ $< L <$ ________, then $\sum a_n$ converges if and only if $\sum b_n$ ________________ .

1m. Ratio Test: $a_n > 0$

Let $\rho = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$.

- If $\rho <$ __________ then $\sum a_n$ converges.
- If $\rho >$ __________ then $\sum a_n$ diverges.
- If $\rho =$ __________ then the test is inconclusive.

1n. Root Test: $a_n > 0$

Let $\rho = \lim_{n\to\infty} (a_n)^{\frac{1}{n}}$.

- If $\rho <$ __________ then $\sum a_n$ converges.
- If $\rho >$ __________ then $\sum a_n$ diverges.
- If $\rho =$ __________ then the test is inconclusive.

1o. Alternating Series Test: $a_n > 0$

If

- a_n __________ a_{n+1} for each $n \in \mathbb{N}$
- $\lim_{n\to\infty} a_n =$ __________

then $\sum(-1)^n a_n$ ______________________________

1p. n^{th}-term test: a_n's are arbitrary

If $\lim_{n\to\infty} a_n \neq 0$ or $\lim_{n\to\infty} a_n$ does not exist, then $\sum a_n$ ______________________________ .

1q. Consider the interval $I = (a - R, a + R)$ center about $x = a$ and of radius R.

Let $y = f(x)$ be a function that can be differentiated N times $x = a$. Then the N^{th}-order Taylor polynomial $y = P_N(x)$ of f about a is (your answer should have a summation sign $\sum$ in it)

$P_N(x) \ =$

1r. Consider the interval $I = (a - R, a + R)$ center about $x = a$ and of radius R.

Let $y = f(x)$ be a function that can be differentieated $N + 1$ times for each $x \in I$.

Consider the the N^{th}-order Taylor Reminder term $R_N(x)$, where $f(x) = P_N(x) + R_N(x)$.

Then an upper bound for $|R_N(x)|$ for an $x \in I$ is:

$|R_N(x)| \leq$

2.

$D_x \ (\cos(\ln x)) \ =$

3.

$D_x \ 7^{(x^2)} \ =$

4. $\int (\tan x)\ \left(\sec^7 x\right)\ dx\ =$

Remark: box your substitution box for more partial credit.

5.

$$\int x^2 \arctan x \, dx \ =$$

Remark: box your substitution box for more partial credit.

6.

$$\int \frac{x^2}{\sqrt{4-x^2}}\, dx \;=$$

Remark: box your substitution box for more partial credit.

7. Let R be the region enclosed by

$$y = x^2 \qquad \text{and} \qquad x = 2 \qquad \text{and} \qquad y = 0\ .$$

Let V be the volume of the solid obtained by revolving the region R about the line $x = 3$.

7a. Make a rough sketch below of the region R, labeling the important points.

7b. Using the disk/washer method, express the volume V as an integral (or maybe 2 integrals). You do NOT have to evaluate the integral(s).

V =

8.

$$\lim_{n\to\infty} \frac{3n^{5/2} + 7n^2 + 9}{-17n^{5/2} + 3n^2 - 9n - 18} =$$

9.

$$\lim_{n\to\infty} \frac{8n^{15} - 7n^{10} + 19}{-5n^{13} + 6n^8 - 6n^5 + 9} =$$

On problems 10 and 11b, check the correct box and then indicte your reasoning below. A correctly checked box without appropriate explanation will receive no points.

10. $\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{n}$

☐ absolutely convergent

☐ conditionally convergent

☐ divergent

11a. Let $a_n = \frac{n^3\ (n!)}{(2n)!}$ Find $\frac{a_{n+1}}{a_n}$. Simplify your answer so that no factorial sign (i.e., !) appears.

answer: $\dfrac{a_{n+1}}{a_n} =$

11b. $\displaystyle\sum_{n=17}^{\infty} (-1)^n \ \frac{n^3\ (n!)}{(2n)!}$

☐ absolutely convergent

☐ conditionally convergent

☐ divergent

12. Consider the formal power series

$$\sum_{n=1}^{\infty} \frac{(2x+8)^n}{n} \ .$$

As we did in class, in the box below draw a diagram indicating for which x's this series is: absolutely convergent, conditionally convergent, and divergent. Of course, indicate your reasoning.

13. Let

$$f(x) = (1 + x)^{3/2}$$

and $a = 0$.

Find the 3rd-order Taylor polynomial of $y = f(x)$ about (or at) $a = 0$.

$P_3(x) \;=$

14. As in problem 13, let

$$f(x) = (1 + x)^{3/2}$$

and $a = 0$.

Let $f(x) = P_3(x) + R_3(x)$, where $y = P_3(x)$ is the 3rd-order Taylor polynomial of $y = f(x)$ about $a = 0$ and $y = R_3(x)$ is the corresponding remainder (i.e., error) term.

Consider the interval $I = (-0.5, 0.5)$ center about $a = 0$. Fix an $x \in I$. Find a good upper bound for $|R_3(x)|$.

$\lvert R_3(x)\rvert \leq$

Remark: you only have to carry out the algebra as far as I indicated in class.