Prof. Girardi	Math 142.003/004	Fall 2005	11.22.05	Exam 3

MARK BOX		
PROBLEM	POINTS	
1	36	
2	6	
3	2	
4	8	
5	8	
6	8	
7	8	
8	8	
9	8	
10	8	
%	100	

NAME: ______________________________

SSN: ______________________________

please check the box of your section below

☐ **Section 003 (MW 9:05 pm)**

or

☐ **Section 004 (MW 10:10 pm)**

INSTRUCTIONS:

(1) To receive credit you must:
  (a) **work in a logical fashion, show all your work, indicate your reasoning**; **no credit will be given for an answer that *just appears***; such explanations help with partial credit
  (b) when applicable put your answer on/in the line/box provided
  (c) if no such line/box is provided, then box your answer
(2) The MARK BOX indicates the problems along with their points. Check that your copy of the exam has all of the problems.
(3) You may **not** use a calculator, books, personal notes.
(4) During this exam, do not leave your seat. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.
(5) This exam covers (from *Calculus* by Anton, Bivens, Davis 8$^{\text{th}}$ ed.): Sections 10.1 – 10.6 .

Problem Inspiration:

1-3. a course handout - you were warned
4-6. homework from § 10.1
7. homework from § 10.3
8-10. homework from § 10.6

Solutions will be available on the course homepage later this afternoon.

For problems 1, 2, and 3, fill in the blanks.

Hint: I do NOT want to see the words absolute nor conditional on this page!

1. For problem 1, let $\sum a_n$ be a **positive-termed** series (i.e. $a_n \geq 0$ for each $n \in \mathbb{N}$).

1a. Integral Test Let $f\colon [1, \infty) \to \mathbb{R}$ be so that

- $a_n = f(\_\_\_\_\_\_\_\_)$ for each $n \in \mathbb{N}$
- f is a ________________________________ function
- f is a ________________________________ function
- f is a ________________________________ function .

Then $\sum a_n$ converges if and only if ________________________________ converges.

1b. Comparison Test

- If $0 \leq a_n \leq b_n$ for all $n \in \mathbb{N}$ and $\sum b_n$ ________________, then $\sum a_n$ ________________.
- If $0 \leq b_n \leq a_n$ for all $n \in \mathbb{N}$ and $\sum b_n$ ________________, then $\sum a_n$ ________________.

1c. Limit Comparison Test Let $b_n > 0$ and $\lim_{n\to\infty} \frac{a_n}{b_n} = L$. If ________ $< L <$ ________, then $\sum a_n$ converges if and only if ________________ .

1d. Ratio Test Let $\rho = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$.

- If $\rho <$ ________ then $\sum a_n$ converges.
- If $\rho >$ ________ then $\sum a_n$ diverges.
- If $\rho =$ ________ then the test is inconclusive.

1e. Root Test Let $\rho = \lim_{n\to\infty} (a_n)^{\frac{1}{n}}$.

- If $\rho <$ ________ then $\sum a_n$ converges.
- If $\rho >$ ________ then $\sum a_n$ diverges.
- If $\rho =$ ________ then the test is inconclusive.

2. For problem 2, we now have an **alternating series**, i.e., $\sum(-1)^n a_n$ where $a_n > 0$ for each $n \in \mathbb{N}$.

Alternating Series Test: If

- a_n ________ a_{n+1} for each $n \in \mathbb{N}$
- $\lim_{n\to\infty} a_n =$ ________

then $\sum(-1)^n a_n$ ________________________

3. For problem 3, we now have an arbitrary series $\sum a_n$ (some terms might be positive, some might be negative, all might be positive, etc ...).

n^{th}**-term test** If $\lim_{n\to\infty} a_n \neq 0$ or $\lim_{n\to\infty} a_n$ does not exist, then $\sum a_n$ ____________ .

4.

$$\lim_{n\to\infty} \frac{4n^3+6n^2-17n+9}{-5n^3+7n^2-9n-18} =$$

5.

$$\lim_{n\to\infty} \frac{7n^2+9}{-5n+2} =$$

Hint: watch your plus and minus.

6.

$$\lim_{n\to\infty} \frac{\ln(n)}{n} =$$

Hint: L'Hopital

7. Geometric Series

7a. If $|r| < 1$, then

$$\sum_{n=0}^{\infty} r^n = \boxed{}$$

Notice (a polite hint for problem 7b), if $|r| <$, then

$$\sum_{\mathbf{n=0}}^{\infty} r^n = \mathbf{1} + r + r^2 + r^3 + r^4 + \ldots .$$

7b. Find the sum of the below series. (Note that the sum begins at $n = 10$ instead of $n = 0$.)

$$\sum_{n=10}^{\infty} \left(\frac{1}{3}\right)^{n-2} = \boxed{}$$

You only have to carry the algebra out as far as I indicated in class.

On problems 8 - 10, check the correct box and then indicte your reasoning below. A correctly checked box without appropriate explanation will receive no points.

8. $\sum_{n=1}^{\infty} (-1)^n \left(\frac{\pi}{e}\right)^n$

☐ absolutely convergent

☐ conditionally convergent

☐ divergent

Hint: $\frac{\pi}{e} \approx \frac{3.14}{2.7} \approx 1.16.$

9. $\displaystyle\sum_{n=17}^{\infty} (-1)^n \; \frac{1}{n!}$

☐ absolutely convergent

☐ conditionally convergent

☐ divergent

10. $\displaystyle\sum_{n=2}^{\infty} (-1)^n \frac{n^2}{n^3+8}$

☐ absolutely convergent

☐ conditionally convergent

☐ divergent

More space for problem 10. $\longrightarrow$

More space for problem 10