Prof. Girardi Math 142.003/004 Fall 2005 10.20.05 Exam 2

MARK BOX		
PROBLEM	POINTS	
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
%	100	

NAME: ______________________________

SSN: ______________________________

please check the box of your section below

☐ **Section 003 (MW 9:05 pm)**

or

☐ **Section 004 (MW 10:10 pm)**

INSTRUCTIONS:

(1) To receive credit you must:
  (a) **work in a logical fashion, show all your work, indicate your reasoning**
  (b) when applicable put your answer on/in the line/box provided
  (c) if no such line/box is provided, then box your answer
(2) The MARK BOX indicates the problems along with their points. Check that your copy of the exam has all of the problems.
(3) You may **not** use a calculator, books, personal notes.
(4) During this exam, do not leave your seat. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.
(5) This exam covers (from *Calculus* by Anton, Bivens, Davis 8^{th} ed.): Sections 8.1 – 8.5, 8.7, 8.8 .

Problem Inspiration:

1. precisely INTEGRAL HANDOUT homework # 35
2. example from class; precisely INTEGRAL HANDOUT homework # 23
3. example from class and homework problem § 8.3 # 3; , precisely problem § 8.3 # 4
4. precisely INTEGRAL HANDOUT homework # 1
5. example from class, homework problem § 8.2 # 1,7; , precisely problem § 8.2 # 4
6. precisely homework problem § 8.4 # 35
7. homework problem § 8.5 # 17; , precisely problem § 8.5 # 18
8. homework problem § 8.7 # 1
9. homework problem § 8.7 # 7
10. homework from 8.8

1. $\boxed{\displaystyle\int \sec^3 x \tan^3 x\, dx = \frac{\sec^5 x}{5} - \frac{\sec^3 x}{3} + C}$

$$\boxed{\begin{aligned} u &= \sec x \\ du &= \sec x \tan x\, dx \end{aligned}}$$

$$\boxed{\begin{gathered} \frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} \\ \Downarrow \\ 1 + \tan^2 x = \sec^2 x \end{gathered}}$$

$$\int \sec^3 x \tan^3 x\, dx = \int \sec^2 x \tan^2 x\, (\sec x \tan x\, dx)$$

$$= \int \sec^2 x\, (\sec^2 x - 1)\, (\sec x \tan x\, dx)$$

$$= \int u^2 (u^2 - 1)\, du$$

$$= \int (u^4 - u^2)\, du$$

$$= \frac{u^5}{5} - \frac{u^3}{3} + C$$

2. $\boxed{\int \ln(1+x)\,dx = (1+x)\ln(1+x) - x + C \overset{\text{or}}{=} x\ln(1+x) - x + \ln(1+x) + C}$

WAY #1

$$\boxed{\begin{array}{ll} u = \ln(1+x) & dv = dx \\ du = \dfrac{dx}{1+x} & v = x \end{array}}$$

$$\int \ln(1+x)\,dx = x\ln(1+x) - \int \frac{x}{1+x}\,dx$$

$$\overset{\text{L.D.}}{=} x\ln(1+x) - \int \left(1 - \frac{1}{1+x}\right) dx$$

$$= x\ln(1+x) - x + \ln(1+x) + C$$

$$= (1+x)\ln(1+x) - x + C$$

WAY #2 - easier!

$$u = \ln(1+x) \qquad dv = dx$$

$$du = \frac{dx}{1+x} \qquad v = 1+x$$

$$\int \ln(1+x)\,dx = (1+x)\ln(1+x) - \int (1+x)\frac{dx}{(1+x)}$$

$$= (1+x)\ln(1+x) - \int dx$$

$$= (1+x)\ln(1+x) - x + C$$

3. $\boxed{\int \cos^2(3x)\,dx = \frac{x}{2} + \frac{1}{12}\sin(6x) + C}$

$$\int \cos^2(3x) = \int \frac{1+\cos 6x}{2}\,dx$$

$$= \frac{1}{2}\left[\int dx + \frac{1}{6}\int \cos 6x\,(6\,dx)\right]$$

$$= \frac{1}{2}\left[x + \frac{1}{6}\sin 6x\right] + C$$

4. $\boxed{\displaystyle\int \frac{dx}{\sqrt{x}\,(1+x)} = 2\tan^{-1}\sqrt{x} + C}$

Hint: $1 + x = 1 + (\sqrt{x}\,)^2$

$$\boxed{\begin{array}{lcl} u = x^{1/2} & \longrightarrow & du = \frac{1}{2}x^{-1/2}\,dx \\ & \text{or} & \\ u^2 = x & \longrightarrow & 2u\,du = dx \end{array}}$$

$$\int \frac{dx}{\sqrt{x}\,(1+x)} = \int \frac{1}{1+(\sqrt{x})^2}\,(x^{-1/2}\,dx)$$

$$= \int \frac{1}{1+u^2}\,(2\,du) = 2\int \frac{du}{1+u^2}$$

$$= 2\tan^{-1} u + C$$

5. $$\boxed{\int x^2e^{(-2x)}\,dx = \frac{-x^2e^{-2x}}{2} - \frac{xe^{-2x}}{2} - \frac{e^{-2x}}{4} + c}$$

$$\boxed{\begin{array}{ll} u_1 = x^2 & dv_1 = e^{-2x}\,dx \\ du_1 = 2x\,dx & v_1 = -\frac{1}{2}e^{-2x} \end{array}} \qquad \boxed{\begin{array}{ll} u_2 = x & dv_2 = e^{-2x}\,dx \\ du_2 = dx & v_2 = -\frac{1}{2}e^{-2x} \end{array}}$$

$$\int x^2 e^{(-2x)}\,dx \overset{①}{=} -\frac{x^2}{2}e^{-2x} - \int -\frac{1}{2}e^{-2x}\,2x\,dx$$

$$= \frac{-x^2}{2}e^{-2x} + \int x e^{-2x}\,dx$$

$$\overset{②}{=} \frac{-x^2}{2}e^{-2x} + \left[-\frac{x}{2}e^{-2x} - \int -\frac{1}{2}e^{-2x}\,dx\right]$$

$$= -\frac{x^2}{2}e^{-2x} - \frac{x}{2}e^{-2x} + \frac{1}{2}\left(-\frac{1}{2}\right)\int e^{-2x}\,(-2\,dx)$$

$$= \frac{-x^2e^{-2x}}{2} - \frac{xe^{-2x}}{2} - \frac{e^{-2x}}{4} + C$$

6. $\boxed{\displaystyle\int \frac{dx}{\sqrt{3+2x-x^2}} = \sin^{-1}\left(\frac{x-1}{2}\right) + c}$

Hint: complete the square.

$$3 + 2x - x^2 = -\left[x^2 - 2x - 3\right]$$

$$= -\left[\underbrace{(x-1)^2}_{x^2-2x+1} \underbrace{-4}_{-4}\right]$$

$$= 4 - (x-1)^2$$

$$\int \frac{dx}{\sqrt{3+2x-x^2}} = \int \frac{dx}{\sqrt{4-(x-1)^2}} = \int \frac{du}{\sqrt{2^2-u^2}}$$

$u = x - 1$

$du = dx$

$$= \sin^{-1}\frac{u}{2} + c$$

$\boxed{a = 2}$

7. $\boxed{\int \frac{x^2}{x^2-3x+2}\,dx = x - \ln|x-1| + 4\ln|x-2| + C}$

Hint: bigger bottoms? $\rightarrow$ NO so do long division

$$\begin{array}{r|l} & \quad 1 \\ x^2-3x+2 & x^2+0x+0 \\ & -(x^2-3x+2) \\ \hline & \quad 3x-2 \end{array}$$

$$\Rightarrow \frac{x^2}{x^2-3x+2} = 1 + \frac{3x-2}{x^2-3x+2}$$

$$\frac{3x-2}{x^2-3x+2} = \frac{3x-2}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2} = \frac{A(x-2)+B(x-1)}{(x-1)(x-2)}$$

$$\Rightarrow 3x-2 = A(x-2) + B(x-1)$$

equate coeff: x: $3 = A + B$

constant: $-2 = -2A - B$

$\oplus$ $1 = -A \Rightarrow \boxed{A=-1} \Rightarrow \boxed{B=4}$

$$\Rightarrow \int \frac{x^2}{x^2-3x+2}\,dx = \int \left[1 + \frac{-1}{x-1} + \frac{4}{x-2}\right] dx$$

8. Use the trapezoidal rule, with $n = 6$ subintervals, to approximate the integral

$$\int_2^5 \sqrt{x-1}\,dx \; .$$

ANSWER: T_6 is equal to:

$$\frac{1}{4}\left[1 + 2\sqrt{1.5} + 2\sqrt{2} + 2\sqrt{2.5} + 2\sqrt{3} + 2\sqrt{3.5} + \sqrt{4}\right]$$

You only have to do arithmetic as far as I indicated in class.

$$T_n = \frac{b-a}{2n}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)\right]$$

$$\Delta x = \frac{b-a}{n} = \frac{5-2}{6} = \frac{3}{6} = \frac{1}{2} = .5$$

x_0	x_1	x_2	x_3	x_4	x_5	x_6
2	$2.5 = \frac{5}{2}$	$3 = \frac{6}{2}$	$3.5 = \frac{7}{2}$	$4 = \frac{8}{2}$	$4.5 = \frac{9}{2}$	5

$$T_6 = \underbrace{\frac{5-2}{2(6)}}_{=\frac{1}{4}}\left[\sqrt{2-1} + 2\sqrt{2.5-1} + 2\sqrt{3-1} + 2\sqrt{3.5-1} + 2\sqrt{4-1} + 2\sqrt{4.5-1} + \sqrt{5-1}\right]$$

9. Recall the Trapezoidal Error Bound Theorem: If f'' is continuous on $[a,b]$ and if K_2 is the maximum value of $|f''(x)|$ on $[a,b]$, then

$$\left| T_n - \int_a^b f(x)\,dx \right| \leq \frac{(b-a)^3 K_2}{12n^2}. \tag{11}$$

Use formula (11) to find an upper bound on the error in problem 8.

Answers:

$K_2 = \frac{1}{4}$
$\left| T_6 - \int_2^5 \sqrt{x-1}\,dx \right| \leq \frac{1}{64}$

You only have to do arithmetic as far as I indicated in class.

$$\left.\begin{aligned} f(x) &= (x-1)^{1/2} \\ f'(x) &= \tfrac{1}{2}(x-1)^{-1/2} \\ f''(x) &= -\tfrac{1}{4}(x-1)^{-3/2} \end{aligned}\right] \Rightarrow \quad K_2 = \max_{2 \le x \le 5} \left| -\tfrac{1}{4}\cdot(x-1)^{-3/2} \right|$$

$$= \frac{1}{4} \max_{2 \le x \le 5} \frac{1}{(x-1)^{3/2}}$$

$$\overset{x=2}{=} \frac{1}{4} \frac{1}{(2-1)^{3/2}} = \frac{1}{4}\cdot\frac{1}{1^{3/2}} = \frac{1}{4}$$

$$\left| T_6 - \int_2^5 \sqrt{x-1}\,dx \right| \le \frac{(5-2)^3}{12\,(6)^2}\cdot\frac{1}{4}$$

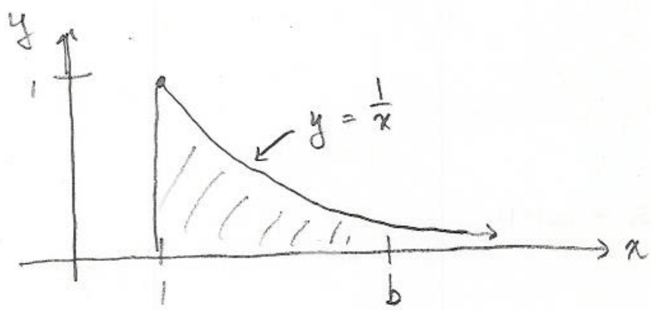
$$= \boxed{\frac{3^3}{12\,(6)^2\cdot 4}}$$ ← to here is ok

$$= \frac{3^3}{3\cdot 4\cdot 3\cdot 2\cdot 3\cdot 2\cdot} \cdot \frac{1}{4}$$

$$= \frac{1}{4\cdot 4}\cdot\frac{1}{4} = \frac{1}{4^3} = \frac{1}{64}$$

10. $\boxed{\displaystyle\int_1^\infty \frac{dx}{x} = \quad \infty \quad \text{or [does not exist]}.}$

Work logically and clearly so that I can see that you see what is going on. Explain your answer.



$$\int_1^\infty \frac{dx}{x} = \lim_{b\to\infty} \int_1^b \frac{dx}{x}$$

$$= \lim_{b\to\infty} \left[\ln x \Big|_{x=1}^{x=b} \right]$$

$$= \lim_{b\to\infty} \left[\ln b - \underbrace{\ln 1}_{=0} \right]$$

$$= \lim_{b\to\infty} \ln b$$

$$= \infty$$