Solutions

1. Use the Fundamental Theorem of Calculus to find:

$$\frac{d}{dx}\int_{17}^{x} e^{\sqrt{t+5}}\,dt = \boxed{e^{\sqrt{x+5}}}\,.$$

2. Evaluate the below derivative.

$$\frac{d}{dx}\,7^{(x^2)} = \boxed{\left(7^{x^2}\right)(\ln 7)\,(2x)}\,.$$

3. Evaluate the below integral. (Hint: factor $3x+6$.)

$$\int_{x=1}^{x=2} \frac{3x+6}{\sqrt{x^2+4x+7}}\,dx = \boxed{3\left(\sqrt{19}-\sqrt{12}\right)}$$

$$\int_{x=1}^{x=2} \frac{3x+6}{\sqrt{x^2+4x+7}}\,dx = 3\int_{x=1}^{x=2} (x^2+4x+7)^{-1/2}\,(x+2)\,dx$$

$$\boxed{\begin{aligned} u &= x^2+4x+7 \\ du &= (2x+4)\,dx \\ \tfrac{1}{2}\,du &= (x+2)\,dx \end{aligned}}$$

$$= \frac{3}{2}\int_{u=12}^{u=19} u^{-1/2}\,du$$

$$= \frac{3}{2}\cdot\frac{2}{1}\,u^{1/2}\Big|_{u=12}^{u=19} = 3\left[\sqrt{19}-\sqrt{12}\right]$$

$$\underline{\text{or}}\ \ 3\left[\sqrt{19}-2\sqrt{3}\right]$$

4. Evaluate the below integral. Your answer should be a real number and should not involve any log, trig, etc, functions.

$$\int_{x=\frac{-1}{\sqrt{3}}}^{x=0} \frac{dx}{1+9x^2} = \boxed{\frac{\pi}{9}}$$

$$\int_{x=-\frac{1}{\sqrt{3}}}^{x=0} \frac{dx}{1+9x^2} = \int_{x=-\frac{1}{\sqrt{3}}}^{x=0} \frac{dx}{1+(3x)^2} = \frac{1}{3}\int_{u=-\frac{3}{\sqrt{3}}}^{u=0} \frac{du}{1^2+u^2}$$

$u = 3x$

$du = 3\,dx$

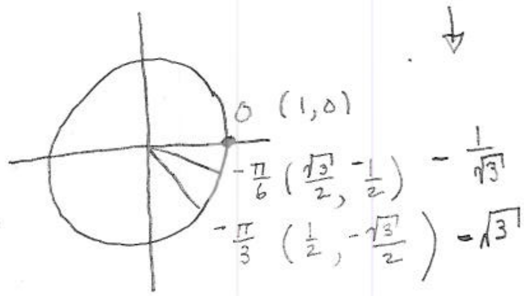
$$= \frac{1}{3} \tan^{-1} \Big|_{u=-\frac{3}{\sqrt{3}}}^{u=0} \qquad -\frac{3}{\sqrt{3}} = -\frac{3}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = -\sqrt{3}$$

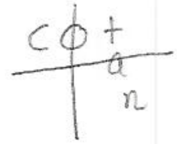
$$= \frac{1}{3}\left[\tan^{-1} 0 - \tan^{-1}(-\sqrt{3})\right]$$

$$= \frac{1}{3}\left[0 - \frac{-\pi}{3}\right]$$

$$= \frac{\pi}{9}$$

$\tan^{-1} x$





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5. The Logarithmic Function Time.

5a. Fill in the one box. By definition, for $x > 0$,

$$\ln x = \int_1^{\boxed{x}} \frac{dt}{t}.$$

5b. Let $\ln a = 3$ and $\ln c = 6$. Then

$$\int_1^{\frac{a}{c}} \frac{dt}{t} = \boxed{-3}.$$

Your answer should be a number and should not have a ln, a, or c in it.

$$\int_1^{a/c} \frac{dt}{t} = \ln\left(\frac{a}{c}\right) = \ln a - \ln c = 3 - 6 = -3$$

(5) This exam covers (from *Calculus* by Anton, Bivens, Davis 8^{th} ed.): Section 6.6, 6.8, 6.9, 7.1 – 7.4, 7.6, 7.7 .

Problem Inspiration:

1. homework problem § 6.6 # 55
2. homework problem § 6.9 # an additional problem
3. homework problem § 6.8 # 33
4. homework problem § 6.8 # 47
5. homework problem § 6.9 # 3
6. homework problem § 7.1 # 31 , actually problem § 7.1 # 32
7. homework problem § 7.2 # 7 , actually problem § 7.2 # 10
8. homework problem § 7.3 # 5 , actually problem § 7.3 # 6
9. homework problem § 7.4 # 7 , actually problem § 7.4 # 8
10. combines ideas from § 7.2 and 7.3

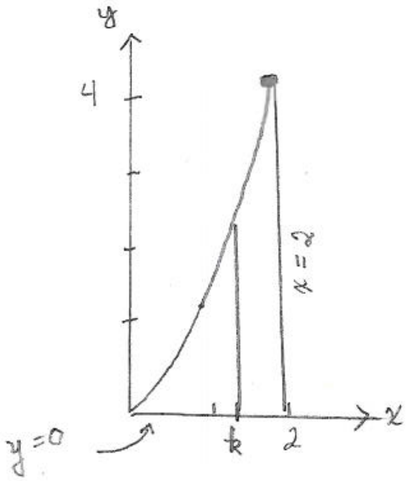
6. Find a vertical line $x = k$ that divides the area enclosed by

$$x = \sqrt{y} \quad \text{and} \quad x = 2 \quad \text{and} \quad y = 0$$

$\Updownarrow$

into two equal parts. $x^2 = y$

ANSWER: the vertical line is $x =$ $\sqrt[3]{4}$.



WANT $\int_0^k x^2\,dx = \int_k^2 x^2\,dx$

$\Leftrightarrow \frac{1}{3}x^3 \Big|_{x=0}^{x=k} = \frac{1}{3}x^3 \Big|_{x=k}^{x=2}$

$\Leftrightarrow \frac{1}{3}(k^3 - 0^3) = \frac{1}{3}(2^3 - k^3)$

$\Leftrightarrow k^3 = 8 - k^3$

$\Leftrightarrow 2k^3 = 8$

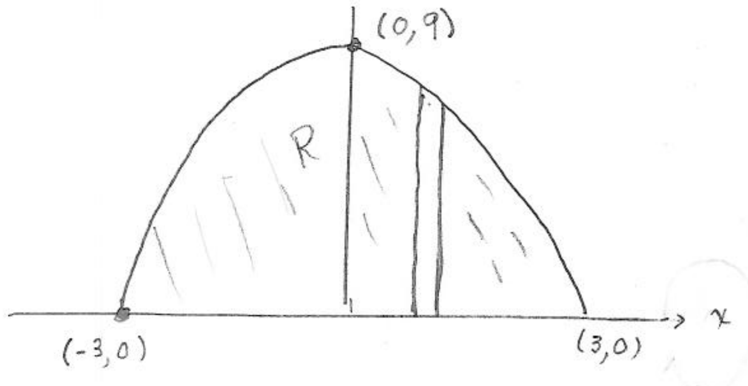
$\Leftrightarrow k^3 = 4$

7. Let R be the region enclosed by

$$y = 9 - x^2 \qquad \text{and} \qquad y = 0\ .$$

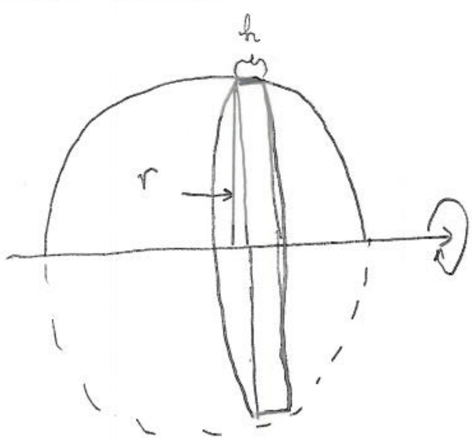
Let V be the volume of the solid obtained by revolving the region R about the x-axis.

7a. Make a rough sketch of the region R, labeling the important points.



7b. Using the disk/washer method, express the volume V as an integral (or maybe 2 integrals). You do NOT have to evaluate the integral(s).

$$V = \int_{x=-3}^{x=3} \pi \left(9 - x^2\right)^2 dx$$



V typical element

= V. of "tuna can" or disk

$= \pi r^2 h$

$= \pi \left(9 - x^2\right)^2 \Delta x$

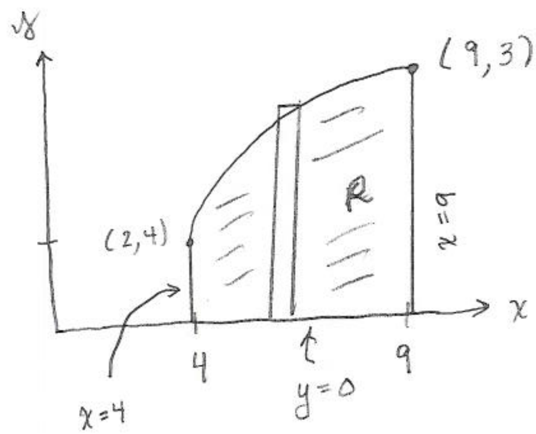
6

8. Let R be the region enclosed by

$$y = \sqrt{x} \quad \text{and} \quad x = 4 \quad \text{and} \quad x = 9 \quad \text{and} \quad y = 0\ .$$

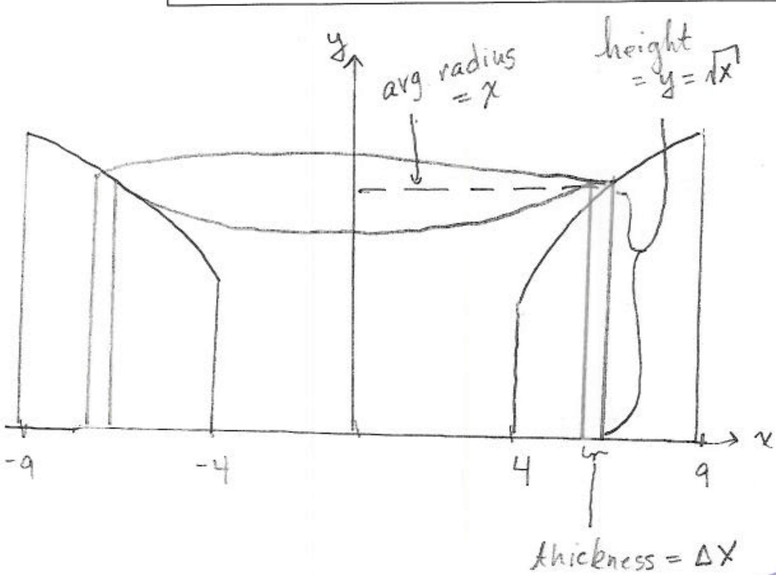
Let V be the volume of the solid obtained by revolving the region R about the y-axis.

8a. Make a rough sketch of the region R, labeling the important points.



8b. Using the cylindrical shell method, express the volume V as an integral (or maybe 2 integrals). You do NOT have to evaluate the integral(s).

$$V = \int_{x=4}^{x=9} 2\pi\, x^{3/2}\, dx$$



V of typical element

= V of shell

= 2π (average radius)(thickness)(height)

$= 2\pi\, x\, \Delta x\, \sqrt{x}$

$= 2\pi\, x\sqrt{x}\, \Delta x$

$\overset{\text{or}}{=} 2\pi\, x^{3/2}\, \Delta x$

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9. Let L be the arc length of the curve

$$x = \frac{y^4}{8} + \frac{y^{-2}}{4} \qquad \text{from} \qquad y = 1 \qquad \text{to} \qquad y = 4\,.$$

Express L as an integral You do NOT have to evaluate the integral.

ANSWER: L= $\displaystyle\int_{y=1}^{y=4} \sqrt{1 + \left(\tfrac{1}{2}y^3 - \tfrac{1}{2}y^{-3}\right)^2}\, dy$

$g(x) = \frac{1}{8}y^4 + \frac{1}{4}y^{-2}$

$g'(x) = \frac{1}{2}y^3 - \frac{1}{2}y^{-3}$

$[g'(x)]^2 = \frac{y^6}{4} - \frac{1}{2} + \frac{y^{-6}}{4}$

$1 + [g'(x)]^2 = \frac{y^6}{4} + \frac{1}{2} + \frac{y^{-6}}{4} = \frac{1}{4}\left(y^3 + y^{-3}\right)^2$

$\sqrt{1 + [g'(x)]^2} = \frac{1}{2}\left(y^3 + y^{-3}\right)$

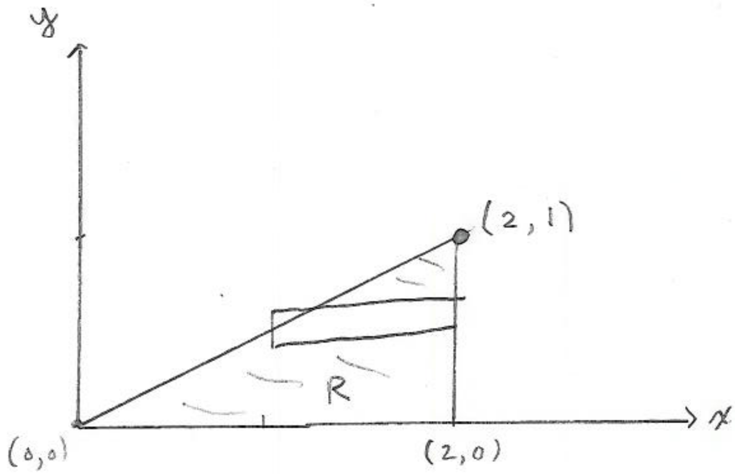
8

10. Let R be the region enclosed by

$$y = \frac{x}{2} \qquad \text{and} \qquad x = 2 \qquad \text{and} \qquad y = 0\ .$$

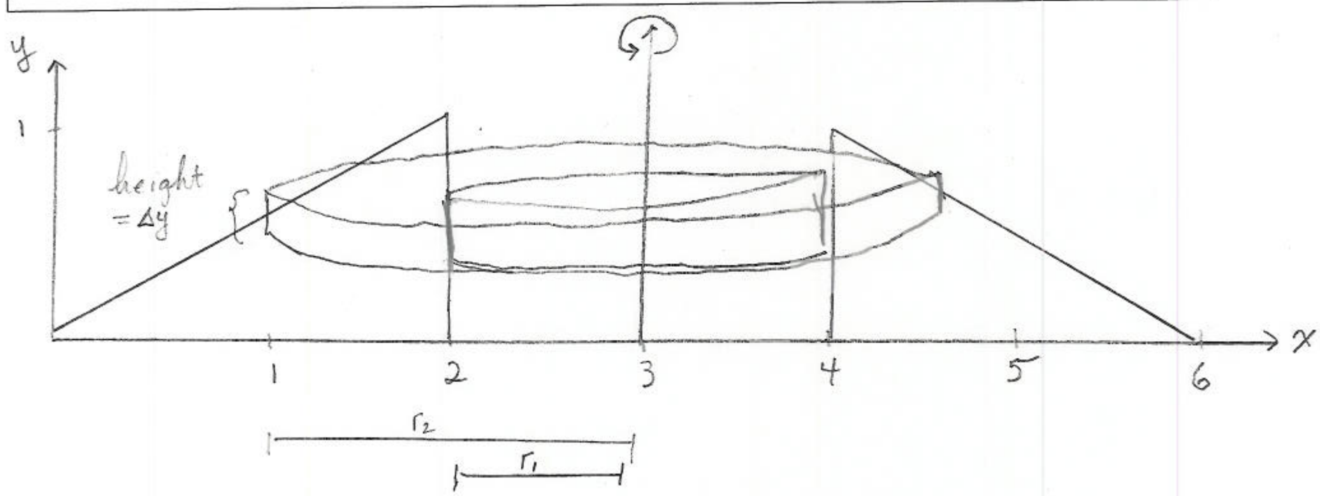
Let V be the volume of the solid obtained by revolving the region R about the line $x = 3$.

10a Make a rough sketch of the region R, labeling the important points.



10b Using the ~~disk~~/washer method, express the volume V as an integral (or maybe 2 integrals). You do NOT have to evaluate the integral(s).

$$\boxed{V = \int_{y=0}^{y=1} \pi \left[(3-2y)^2 - 1^2 \right] dy \ \overset{\text{or}}{=}\ \pi \int_{y=0}^{y=1} (4y^2 - 12y + 8)\, dy}$$



Volume of typical element $= \pi (r_2^2 - r_1^2)$ (height)

$= \pi \left((3-2y)^2 - 1^2 \right) \Delta y$