1. $D_x\left[e^{3x^2+1}\right] = 6x\, e^{3x^2+1}$

2. $D_x\left[\ln(3x^2+17)\right] = \dfrac{6x}{3x^2+17}$

Variation of Exam 1 #4

3. $D_x\left((1+x)^{2x}\right) = \left[2\ln(1+x) + \dfrac{2x}{1+x}\right](1+x)^{2x}$

HINT: Use logarithmic differentiation.

$y = (1+x)^{2x}$

$\ln y = 2x \ln(1+x)$

$\dfrac{1}{y}\dfrac{dy}{dx} = 2\ln(1+x) + \dfrac{2x}{1+x}$

Variation of Exam 1 #5

6. $\int \sin^2 x\,dx = \dfrac{x}{2} - \dfrac{\sin 2x}{4} + C$

textbook Example 2 §8.2

8. $\int \ln x\,dx = x\ln x - x + C$

textbook §8.4 Example 2

9. $\int x^2 \sin x\,dx = -x^2\cos x + 2x\sin x + 2\cos x + C$

textbook §8.4 Example 4

MATH 142 - Spring Final

Prof. Girardi

4. The rate of decay of a radioactive substance is proportional to the amount of such substance present. Today we have 30 grams of a radioactive substance. Given that one-third of the substance decays every 5 years, how much will be left t years from today? *Clearly explain your notation.*

ANSWER: $P(t) = 30\, e^{\frac{1}{5}\ln\left(\frac{2}{3}\right) t}$ grams

HINT: your answer should have a t in it.

Let t_0 = initial time = today

t = the # of years after t_0

$P(t)$ = # of grams of the radioactive substance at time t

$1 \Rightarrow \dfrac{dP}{dt} = kP(t) \Rightarrow P(t) = P_0 e^{kt} \Rightarrow \boxed{P(t) \overset{3}{=} 30e^{kt}}$

$\frac{2}{3}(30) \overset{2}{=} P(5) \overset{3}{=} 30e^{k(5)}$

$\Rightarrow e^{5k} = \frac{2}{3}$

$\Rightarrow \ln e^{5k} = \ln\left(\frac{2}{3}\right)$

$\Rightarrow 5k = \ln\left(\frac{2}{3}\right)$

$\Rightarrow k = \frac{1}{5}\ln\left(\frac{2}{3}\right)$

Variation of Exam 1 #10

5. $\int \sin^3 x\,dx = -\int (1-\cos^2 x)(-\sin x)\,dx = -\int (1-u^2)\,du = \dfrac{u^3}{3} - u + C = \dfrac{\cos^3 x}{3} - \cos x + C$

$u = \cos x$

10. $\int \frac{x^4-2x^2+4x+1}{x^3-x^2-x+1}\,dx = \frac{x^2}{2} + x + \ln(x-1) - 2(x-1)^{-1} - \ln(x+1) + C$

HINT: $x^3-x^2-x+1=(x-1)^2(x+1)$

$$\frac{x^4-2x^2+4x+1}{x^3-x^2-x+1} \overset{LD}{=\!=} (x+1) + \frac{4x}{x^3-x^2-x+1}$$

$$\frac{4x}{(x-1)^2(x+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1}$$

$$\Rightarrow \quad 4x = A(x-1)(x+1) + B(x+1) + C(x-1)^2$$

$\boxed{x=1} \Rightarrow 4 = 2B \Rightarrow \boxed{B=2}$

$\boxed{x=-1} \Rightarrow -4 = 4C \Rightarrow \boxed{C=-1}$

$x^2 : 0 = A + C \implies A = +1$

x :

x^0 : not needed

$$\Rightarrow \frac{4x}{(x-1)^2(x+1)} = \frac{1}{x-1} + \frac{2}{(x-1)^2} + \frac{-1}{x+1}$$

$$\frac{2}{(x-1)^2} = 2(x-1)^{-2}$$

11. $\lim_{x\to\infty} x^{\frac{1}{x}} = e^0 = 1$

Exam 2 # 8

12. $\int_0^{\infty} \frac{dx}{x+1} = \infty$ or DNE or diverges

textbook § 9.5 # 23

13. $\lim_{n\to\infty} \frac{12n^{17}+188n^7-19n}{4n^{18}-n^9+10} = \lim_{n\to\infty} \frac{\frac{12}{n}+\frac{188}{n^{11}}-\frac{19}{n^{17}}}{4-\frac{1}{n^9}+\frac{10}{n^{18}}} = \frac{0+0-0}{4-0+0} = 0$

divide thru by n^{18}

Exam 3 # 8

14. $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}}$

- ☐ absolutely convergent
- ☒ conditionally convergent
- ☐ divergent

Serious Series # 14

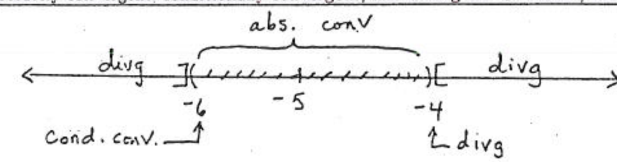
15. $\sum_{n=1}^{\infty} (-1)^n \frac{(3^n)\,n!}{(2n)!}$

- ☒ absolutely convergent
- ☐ conditionally convergent
- ☐ divergent

16. Consider the formal power series $\sum_{n=1}^{\infty} \frac{(x+5)^n}{n}$.

An Example from Class

As we did in class, in the box below draw a diagram indicating for which x's this series is: absolutely convergent, conditionally convergent, and divergent. Of course, indicate your reasoning.



7. $\int \frac{x^3}{\sqrt{1-x^2}}\,dx = \int \frac{\sin^3\theta}{\cos\theta}\cdot\cos\theta\,d\theta = \int \sin^3\theta\,d\theta$

$x=\sin\theta$
(Trig Subst.)

$$\overset{\#5}{=} \frac{\cos^3\theta}{3} - \cos\theta + C$$

$$= \frac{\left[(1-x^2)^{1/2}\right]^3}{3} - \sqrt{1-x^2} + C$$

$$= \boxed{\frac{(1-x^2)^{3/2}}{3} - \sqrt{1-x^2} + C}$$

17. The third order Taylor polynomial of

$$f(x) = (1+x)^{\frac{3}{2}}$$

about $a = 0$ is:

Homework §10.8 # 17
but made easier

$P_3(x) = 1 + \frac{3}{2}x^1 + \frac{1}{2!}\frac{3}{4}x^2 - \frac{1}{3!}\frac{3}{8}x^3 \overset{\text{or}}{=} 1 + \frac{3x}{2} + \frac{3x^2}{8} - \frac{x^3}{16}$

$f(x) = (1+x)^{3/2}$ $\qquad f(0) = 1$

$f^{(1)}(x) = \frac{3}{2}(1+x)^{1/2}$ $\qquad f^{(1)}(0) = \frac{3}{2}$

$f^{(2)}(x) = \frac{3}{2}\cdot\frac{1}{2}(1+x)^{-1/2}$ $\qquad f^{(2)}(0) = \frac{3}{4}$

$f^{(3)}(x) = \frac{3}{2}\cdot\frac{1}{2}\cdot\frac{-1}{2}(1+x)^{-3/2}$ $\qquad f^{(3)}(0) = \frac{-3}{8}$

A simplification of HMWK § 11.1 # 33

18. Consider the function $f(x) = \ln(2+x)$. Let $a = 0$. We know that we can write the function as

$$f(x) = P_3(x) + R_3(x)$$

where P_3 is the third order Taylor polynominal of f about $a = 0$ and R_3 is the corresponding remainder term.

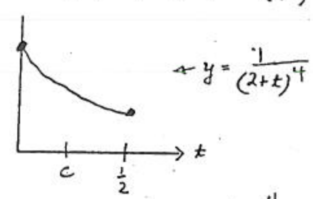
18a Find a formula for the remainder term $R_3(x)$. Your answer should have a "c" in it, be sure to indicate where c lies.

$R_3(x) = \frac{-6(2+c)^{-4}}{4!}x^4 \overset{\text{or}}{=} \frac{-x^4}{4(2+c)^4}$

where c is between x and 0

$f(x) = \ln(2+x) \Rightarrow f^{(4)}(x) = -6(2+x)^{-4}$

$\frac{1}{(0.5)^4} = \frac{1}{2^4}$



$|R_3(0.5)| = \frac{(0.5)^4}{4}\cdot\frac{1}{(2+c)^4} \le \frac{(0.5)^4}{4}\cdot\frac{1}{(0.5)^4} = \frac{1}{4}$

18b Find a good upper bound for $|R_3(0.5)|$. Your answer should not have a "c" in it but you do not have to do arithmetic. **Do you work on the back of the previous page.**

$|R_3(0.5)| \le \frac{1}{4}$

HM §12.8 # 11

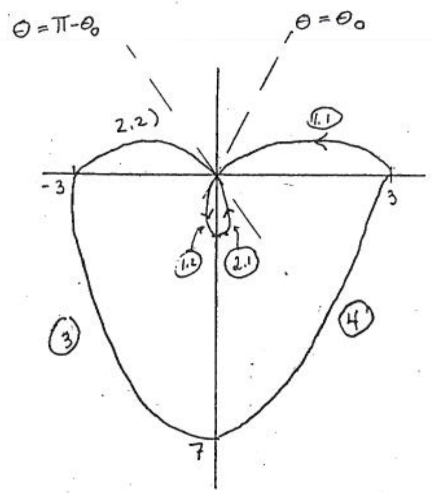
19. Using the method from class, sketch the graph of the polar equation $r = 3 - 4\sin\theta$.

$\frac{\text{period}}{4} = \frac{2\pi/1}{4} = \frac{\pi}{2}$ || $0 = 3 - 4\sin\theta \iff \sin\theta = \frac{3}{4}$

$\theta_0 = \sin^{-1}\frac{3}{4}$

θ	$\sin\theta$	$r = 3-4\sin\theta$
$0 \xrightarrow{1} \frac{\pi}{2}$	$0 \to 1$	$3 \to -1$
$\frac{\pi}{2} \xrightarrow{2} \pi$	$1 \to 0$	$-1 \to 3$
$\pi \xrightarrow{3} \frac{3\pi}{2}$	$0 \to -1$	$3 \to 7$
$\frac{3\pi}{2} \xrightarrow{4} 2\pi$	$-1 \to 0$	$7 \to 3$

θ	r
$0 \xrightarrow{1.1} \theta_0$	$3 \to 0$
$\theta_0 \xrightarrow{1.2} \frac{\pi}{2}$	$0 \to -1$
$\frac{\pi}{2} \xrightarrow{2.1} \pi - \theta_0$	$-1 \to 0$
$\pi - \theta_0 \xrightarrow{2.2} \pi$	$0 \to 3$



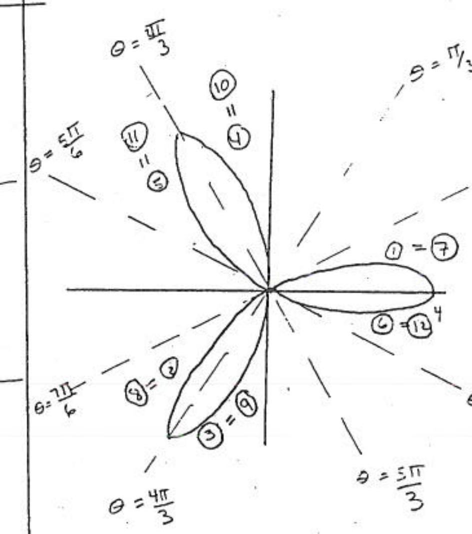
HMWK §12.8 # 15 $\frac{\text{period}}{4} = \frac{3}{4} =$

20. Consider the polar equation $r = 4\cos(3\theta)$.

20a Using the method from class, sketch the graph of this polar equation.

Make your chart on the back of the previous page.

θ	3θ	$r = 4\cos 3\theta$
$0 \xrightarrow{1} \frac{\pi}{6}$	$0 \to \frac{\pi}{2}$	$4 \to 0$
$\frac{\pi}{6} \xrightarrow{2} \frac{\pi}{3}$	$\frac{\pi}{2} \to \pi$	$0 \to -4$
$\frac{\pi}{3} \xrightarrow{3} \pi/2$	$\pi \to \frac{3\pi}{2}$	$-4 \to 0$
$\frac{\pi}{2} \xrightarrow{4} \frac{2\pi}{3}$	$\frac{3\pi}{2} \to 2\pi$	$0 \to 4$
$\frac{2\pi}{3} \xrightarrow{5} \frac{5\pi}{6}$		$4 \to 0$
$\frac{5\pi}{6} \xrightarrow{6} \pi$		$0 \to -4$
$\pi \xrightarrow{7} \frac{7\pi}{6}$	repeat	$-4 \to 0$
$\frac{7\pi}{6} \xrightarrow{8} \frac{4\pi}{3}$		$0 \to 4$
$\frac{4\pi}{3} \xrightarrow{9} \frac{3\pi}{2}$		$4 \to 0$
$\frac{3\pi}{2} \xrightarrow{10} \frac{5\pi}{3}$	repeat	$0 \to -4$
$\frac{5\pi}{3} \xrightarrow{11} \frac{11\pi}{6}$		$-4 \to 0$
$\frac{11\pi}{6} \xrightarrow{12} 2\pi$		$0 \to 4$



20b Express the area enclosed by this polar equation as an integral (but you do not have to evaluate this integral).

$$\boxed{\text{Area} = 6 \cdot \frac{1}{2} \int_{\theta=0}^{\theta=\frac{\pi}{6}} (4\cos 3\theta)^2 \, d\theta \overset{\text{or}}{=} 48 \int_0^{\pi/6} \cos^2(3\theta)\, d\theta}$$

$6 \cdot \frac{1}{2} \cdot 4^2 = 3 \cdot 16$