Prof. Girardi		Math $142.001/002$		Spring 2003	04.06.03	Final Exam
MARK BOX						
PROBLEM	POINTS					
1	5					
2	5					
3	5					
4	5					
5	5		NAME):		
6	5					
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8	5		SSN:			
9	5					
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11	5		Sectior	n 001 (MW 9:05)		
12	5		2000101			
13	5			or		
14	5		a			
15	5		Section	1002 (MW 10:10)))	
16	5					
17	5					
18	5					
19	5					
20	5					
%	100					

INSTRUCTIONS:

- (1) To receive credit you must:
 - (a) work in a logical fashion, show all your work, indicate your reasoning
 - (b) justify your reasoning
 - (c) when applicable put your answer on/in the line/box provided
 - (d) if no such line/box is provided, then box your answer.
- (2) The MARK BOX indicates the problems along with their points. Check that your copy of the exam has all of the problems.
- (3) You may **not** use a calculator, books, personal notes. Give exact answers: for example, write ln 2 instead of .6931, write $\sqrt{2}$ instead of 1.414, write π instead of 3.1415, write $\frac{1}{3}$ instead of 0.3333.
- (4) During this exam, do not leave your seat. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.
- (5) This exam covers (from *Calculus* by Varberg, Purcell, Rigdon, 8th ed.): Chs 7-10, Sections 11.1, 12.6-12.8.

$$D_x \left[e^{3x^2 + 1} \right] =$$

$$D_x \left[\ln \left(3x^2 + 17 \right) \right] =$$

$$D_x \left((1+x)^{2x} \right) =$$

HINT: Use logarithmic differentiation.

4. The rate of decay of a radioactive substance is proportional to the amount of such substance present. Today we have 30 grams of a radioactive substance.

Given that one-third of the substance decays every 5 years, how much will be left t years from today? Clearly explain your notation.

ANSWER: grams

HINT: your answer should have a t in it.

 $\int \sin^3 x \, dx =$

•

$$\int \sin^2 x \, dx =$$

HINT: need a trig identity.

$$\int \frac{x^3}{\sqrt{1-x^2}} \, dx = +C$$

 $\int \ln x \, dx =$

J

$$\int x^2 \sin x \, dx =$$

$$\int \frac{x^4 - 2x^2 + 4x + 1}{x^3 - x^2 - x + 1} \, dx = +C$$

HINT: $x^3 - x^2 - x + 1 = (x - 1)^2(x + 1)$

 $\lim_{x \to \infty} x^{\frac{1}{x}} =$

$$\int_0^\infty \frac{dx}{x+1} =$$

 $\lim_{n \to \infty} \quad \frac{12n^{17} + 188n^7 - 19n}{4n^{18} - n^9 + 10} \quad = \quad$

 $\hfill\square$ absolutely convergent

14.
$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}}$$

 $\hfill\square$ conditionally convergent

 \Box divergent

15.
$$\sum_{n=1}^{\infty} (-1)^n \frac{(3^n) n!}{(2n)!}$$

- $\hfill\square$ absolutely convergent
- $\hfill\square$ conditionally convergent
- \Box divergent

16. Consider the formal power series

$$\sum_{n=1}^{\infty} \frac{\left(x+5\right)^n}{n} \, .$$

As we did in class, in the box below draw a diagram indicating for which x's this series is: absolutely convergent, conditionally convergent, and divergent. Of course, indicate your reasoning.

17. The third order Taylor polynomial of

$$f(x) = (1+x)^{\frac{3}{2}}$$

about a = 0 is:

 $P_3(x) =$

18. Consider the function $f(x) = \ln (2 + x)$. Let a = 0. We know that we can write the function as

$$f(x) = P_3(x) + R_3(x)$$

where P_3 is the third order Taylor polynomial of f about a = 0 and R_3 is the corresponding remainder term.

18aF ind a formula for the remainder term $R_3(x)$. Your answer should have a "c" in it, be sure to indicate where c lies.

 $R_3(x) =$

18bE ind a good upper bound for $|R_3(0.5)|$. Your answer should **not** have a "c" in it but you do not have to do arithmetic. **Do you work on the back of the previous page.**

 $|R_3(0.5)| \le$

19. Using the method from class, sketch the graph of the polar equation $r = 3 - 4\sin\theta$.

20. Consider the polar equation $r = 4\cos(3\theta)$.

20 a. Using the method from class, sketch the graph of this polar equation.

Make your chart on the back of the previous page.

20bExpress the area enclosed by this polar equation as an integral (but you do not have to evaluate this integral).

Area =