

MARK BOX		
PROBLEM	POINTS	
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
%	100	

NAME: Key

SSN: \_\_\_\_\_

Section 001 (MW 9:05)

or

Section 002 (MW 10:10)

**INSTRUCTIONS:**

- (1) To receive credit you must:
  - (a) work in a logical fashion, show all your work, indicate your reasoning
  - (b) when applicable put your answer on/in the line/box provided
  - (c) if no such line/box is provided, then box your answer
- (2) The MARK BOX indicates the problems along with their points. Check that your copy of the exam has all of the problems.
- (3) You may not use a calculator, books, personal notes. Give exact answers: for example, write  $\ln 2$  instead of .6931, write  $\sqrt{2}$  instead of 1.414, write  $\pi$  instead of 3.1415, write  $\frac{1}{3}$  instead of 0.3333.
- (4) During this exam, do not leave your seat. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.
- (5) This exam covers (from *Calculus* by Varberg, Purcell, Rigdon, 8<sup>th</sup> ed.): Sections 10.1-10.5.

**ADDITIONAL INSTRUCTIONS**

In problems 1 - 5, find the limit of each sequence.  
 In problems 7 - 10, check the appropriate box of what that series is doing.  
 Always justify your answer.

**Problem Inspiration:**

- 1-3. An idea we have gone over-and-over in class.
- 4-5. examples from class
6. just changed numbers from an example from class
- 7-10. minor variations of problems from **SERIOUS SERIES PROBLEMS**.

$$1. \lim_{n \rightarrow \infty} \frac{12n^2 - 3n + 1}{3n^2 - 17n + 100} = \lim_{n \rightarrow \infty} \frac{12 - \frac{3}{n} + \frac{1}{n^2}}{3 - \frac{17}{n} + \frac{100}{n^2}} = \frac{12 - 0 + 0}{3 - 0 + 0} = \boxed{4}$$

or.

$$\lim_{n \rightarrow \infty} \frac{\boxed{12} n^{\boxed{2}} - 3n + 1}{\boxed{3} n^{\boxed{2}} - 17n + 100} = \frac{12}{3} = 4.$$

same

$$2. \lim_{n \rightarrow \infty} \frac{3n^2 + 4}{n - 1} = \lim_{n \rightarrow \infty} \frac{3 + \frac{4}{n^2}}{\frac{1}{n} - \frac{1}{n^2}} = \frac{\cancel{3} + 0}{0 - 0} = \frac{\cancel{3}}{0} = \boxed{\infty} \text{ or DNE}$$

or diverge

$$3. \lim_{n \rightarrow \infty} \frac{n - 1}{3n^2 + 4} = \lim_{n \rightarrow \infty} \frac{\frac{1}{n} - \frac{1}{n^2}}{3 + \frac{4}{n^2}} = \frac{0 - 0}{3 + 0} = \frac{0}{3} = \boxed{0}$$

$$4. \lim_{n \rightarrow \infty} (0.9999)^n = 0 \quad \text{since } \lim_{n \rightarrow \infty} r^n = \begin{cases} 0 & |r| < 1 \\ 1 & r = 1 \\ \text{DNE} & |r| > 1 \text{ or } r = -1 \end{cases}$$

we did in class

$$5. \lim_{n \rightarrow \infty} \frac{n}{e^n} = 0 \quad \text{since } \lim_{x \rightarrow \infty} \frac{x}{e^x} \stackrel{\text{L'H}}{=} \lim_{x \rightarrow \infty} \frac{1}{e^x} = 0$$

or you can say "since exp. grows so much faster than poly"

$$6. \sum_{n=3}^{\infty} 9 \left(\frac{1}{10}\right)^n = 9 \left[ \left(\frac{1}{10}\right)^3 + \left(\frac{1}{10}\right)^4 + \left(\frac{1}{10}\right)^5 + \dots \right]$$

we want 1 here so factor out  $\left(\frac{1}{10}\right)^3$

$$= 9 \cdot \frac{1}{10^3} \left[ 1 + \left(\frac{1}{10}\right)^1 + \left(\frac{1}{10}\right)^2 + \dots \right] = \frac{9}{10^3} \cdot \frac{1}{1 - \frac{1}{10}} = \frac{9}{10^3} \cdot \frac{10}{9} = \frac{1}{10^2} = \boxed{\frac{1}{100}}$$

$$7. \sum (-1)^n \frac{n+1}{n!} \quad \boxed{\text{abs. conv.}}$$

$$\text{Consider } \sum |(-1)^n \frac{n+1}{n!}| = \sum \frac{n+1}{n!}$$

factor = ratio test

$$\lim_{n \rightarrow \infty} \frac{(n+1)+1}{(n+1)!} \cdot \frac{n!}{n+1} = \lim_{n \rightarrow \infty} \frac{n+2}{n+1} \cdot \frac{n!}{n!(n+1)} = \lim_{n \rightarrow \infty} \frac{n+2}{n+1} \cdot \frac{1}{n+1} = \frac{1}{1} \cdot 0 = 0$$

$$8. \sum_{n=1}^{\infty} (-1)^n \frac{1}{n^{1/2}} \quad \boxed{\text{cond. conv}}$$

Consider  $\sum |(-1)^n \frac{1}{n^{1/2}}| = \sum \frac{1}{n^{1/2}} \rightarrow$  divg p-series,  $p = \frac{1}{2} \leq 1$

AST  $\sum (-1)^n a_n$  where  $a_n = \frac{1}{n^{1/2}}$ , clearly,  $a_n$  decr. &  $\lim_{n \rightarrow \infty} a_n = 0 \Rightarrow \sum (-1)^n a_n$  cond. conv.

$$9. \sum (-1)^n \frac{\sin n}{n^2 + 1} \quad \boxed{\text{abs. conv.}}$$

$$\left| (-1)^n \frac{\sin n}{n^2 + 1} \right| = \frac{|\sin n|}{n^2 + 1} \leq \frac{1}{n^2} \cdot \sum \frac{1}{n^2} \text{ conv (p-series, } p=2 > 1)$$

$$\text{Comparison Test } \Rightarrow \sum |(-1)^n \frac{\sin n}{n^2 + 1}| \text{ conv.}$$

$$10. \sum (-1)^n \frac{n^2 + 1}{3n^2 + n - 1} \quad \boxed{\text{divg}} \text{ by } n^{\text{th}} \text{ term test for divergence,}$$

$\lim_{n \rightarrow \infty} (-1)^n \frac{n^2 + 1}{3n^2 + n - 1} \neq 0$