| Prof. Girardi | Math 142 | Fall 2003 | 12.08 .03 | Final Exam |
| :---: | :---: | :---: | :---: | :---: |
| MARK BOX |  |  |  |  |


| MARK BOX |  |  |
| :---: | :---: | :--- |
| PROBLEM | POINTS |  |
| 1 | 5 |  |
| 2 | 5 |  |
| 3 | 5 |  |
| 4 | 5 |  |
| 5 | 5 |  |
| NAME: |  |  |

SSN: $\qquad$

## Section 007 (MW 12:20 pm)

or

## Section 008 (MW 2:30)

## INSTRUCTIONS:

(1) To receive credit you must:
(a) work in a logical fashion, show all your work, indicate your reasoning
(b) justify your reasoning
(c) when applicable put your answer on/in the line/box provided
(d) if no such line/box is provided, then box your answer.
(2) The mark box indicates the problems along with their points.

Check that your copy of the exam has all of the problems.
(3) You may not use a calculator, books, personal notes. Give exact answers: for example, write $\ln 2$ instead of .6931 , write $\sqrt{2}$ instead of 1.414 , write $\pi$ instead of 3.1415 , write $\frac{1}{3}$ instead of 0.3333 .
(4) During this exam, do not leave your seat. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.
(5) This exam covers (from Calculus by Varberg, Purcell, Rigdon, $8^{\text {th }}$ ed.): Chs 7-10, Sections 11.1, 12.6-12.8 .

2. $\begin{aligned} & \\ & D_{x}[\ln (\cos (4 x))]=\end{aligned}$

4. The rate of decay of a radioactive substance is proportional to the amount of such substance present. Today we have 30 grams of a radioactive substance.
Given that one-fourth of the substance decays every 7 years, how much will be left $t$ years from today? Clearly explain your notation.

ANSWER: grams

HINT: your answer should have a $t$ in it.
5.


8.
9. $\int x^{2} \arctan x d x=+C$

11.

$$
\int \frac{x}{x^{4}+4 x^{2}+8} d x=
$$

12. 

$\int e^{x} \cos x d x=$
13. Let $c$ be some constant with $c \neq 0$.
$\lim _{x \rightarrow \infty}\left(1+\frac{c}{x}\right)^{x}=$
14.
$\int_{0}^{\infty} \frac{d x}{x+1}=$
15.
$\lim _{n \rightarrow \infty} \frac{12 n^{17}+188 n^{7}-19 n}{4 n^{16}-n^{9}+10}=$

## absolutely convergent

16. $\sum_{n=1}^{\infty}(-1)^{n} \frac{1}{\sqrt{n^{2}-1}}$conditionally convergent
divergent
absolutely convergent
17. $\sum_{n=1}^{\infty}(-1)^{n} \frac{(n+1)!}{(2 n)!} \quad \square$ conditionally convergent
divergent
18. Consider the formal power series

$$
\sum_{n=1}^{\infty} \frac{(x+5)^{n}}{n}
$$

As we did in class, in the box below draw a diagram indicating for which $x$ 's this series is: absolutely convergent, conditionally convergent, and divergent. Of course, indicate your reasoning.
19. Consider the function $f(x)=e^{2 x}$. Let $a=0$. We know that we can write the function as

$$
f(x)=P_{3}(x)+R_{3}(x)
$$

where $P_{3}$ is the third order Taylor polynominal of $f$ about $a=0$ and $R_{3}$ is the corresponding remainder term.
19a.Find $P_{3}(x)$.
$P_{3}(x)=$

19bFind a formula for the remainder term $R_{3}(x)$. Your answer should have a " $c$ " in it. Be sure to indicate where $c$ lies.

$$
R_{3}(x)=
$$

19c.Find a good upper bound for $\left|R_{3}(x)\right|$ for $-0.5 \leq x \leq 0.5$. Your answer should not have a " $c$ " in it but you do not have to do arithmetic. Do you work on the back of the previous page.
$\square$
20. Consider the polar equation $r=4 \sin (2 \theta)$.

20a.Using the method from class, sketch the graph of this polar equation.
Make your chart on the back of the previous page.

20bExpress the area enclosed by this polar equation as an integral (but you do not have to evaluate this integral). Use symmetry

