Prof. Girardi	Math 142	Fall 2003	10.23.03	Exam 2

MARK BOX

PROBLEM	POINTS	
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
LSUvsUSC		3
%	100	

NAME: Library

SSN: ______________

Section 007 (MW 12:20 pm)

or

Section 008 (MW 2:30 pm)

INSTRUCTIONS:

(1) To receive credit you must:
  (a) work in a logical fashion, show all your work, indicate your reasoning
  (b) when applicable put your answer on/in the line/box provided
  (c) if no such line/box is provided, then box your answer
(2) The MARK BOX indicates the problems along with their points.
Check that your copy of the exam has all of the problems.
(3) You may **not** use a calculator, books, personal notes. Give exact answers: for example, write $\ln 2$ instead of .6931, write $\sqrt{2}$ instead of 1.414, write π instead of 3.1415, write $\frac{1}{3}$ instead of 0.3333.
(4) During this exam, do not leave your seat. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.
(5) This exam covers (from *Calculus* by Varberg, Purcell, Rigdon, 8th ed.): Chapters 8 and 9 .

Problem Inspiration:

1. class handout of 119 integrals
2. homework problem
3. class handout of 119 integrals
4. homework problem, an example from class, Girardi's Spring 03 exam 2
5. an example from class
6. class handout of 119 integrals, homework problem, an example from class
7. homework problem
8. homework problem, Girardi's Spring 03 exam 2
9. homework problem
10. ~~an example from class~~ homework problem

1.

$$\boxed{\int \sec^3 x \tan^3 x\,dx = \frac{\sec^5 x}{5} - \frac{\sec^3 x}{3} + C}$$

$\cancel{u = \tan x \quad du = \sec^2 x\,dx}$ or $\boxed{u = \sec x \quad du = \sec x \tan x\,dx}$ $\quad \frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$, $\quad 1 + \tan^2 x = \sec^2 x$

$$\int \sec^3 x \tan^3 x\,dx = \int \sec^2 x \tan^2 x\,(\sec x \tan x\,dx) = \int \sec^2 x\,(\sec^2 x - 1)(\sec x \tan x\,dx)$$

$$= \int u^2(u^2-1)\,du = \int (u^4 - u^2)\,du = \frac{u^5}{5} - \frac{u^3}{3} + C$$

2.

$$\boxed{\int \frac{x\,dx}{\sqrt{3x+4}} = \int \frac{u^2-4}{3}\cdot\frac{1}{\cancel{u}}\cdot\frac{2\cancel{u}\,du}{3} = \frac{2}{9}\left(\frac{u^3}{3} - 4u\right) = \frac{2(3x+4)^{3/2}}{27} - \frac{8(3x+4)^{1/2}}{9} + C}$$

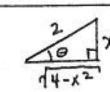
$u = (3x+4)^{1/2} \Rightarrow u^2 = 3x+4 \Rightarrow 2u\,du = 3\,dx$

3.

$$\boxed{\int \frac{x^2\,dx}{\sqrt{4-x^2}} = 2\sin^{-1}\left(\frac{x}{2}\right) - 2\cdot\frac{x}{2}\,\frac{\sqrt{4-x^2}}{2} + C \cong 2\sin^{-1}\left(\frac{x}{2}\right) - \frac{x\sqrt{4-x^2}}{2} + C}$$

$x = 2\sin\theta \longrightarrow \sin\theta = \frac{x}{2}$

$dx = 2\cos\theta\,d\theta$



$\cos\theta = \frac{\sqrt{4-x^2}}{2}$

$\sqrt{4-x^2} = \sqrt{4-4\sin^2\theta} = 2\sqrt{1-\sin^2\theta} = 2\sqrt{\cos^2\theta} = 2\cos\theta$

$$\int \frac{x^2\,dx}{\sqrt{4-x^2}} = \int \frac{(4\sin^2\theta)(2\cos\theta\,d\theta)}{2\cos\theta} = 4\int \sin^2\theta\,d\theta = 4\cdot\frac{1}{2}\int(1-\cos 2\theta)\,d\theta$$

$$= 2\int d\theta - 2\cdot\frac{1}{2}\int \cos 2\theta\,(2\,d\theta) = 2\theta - \sin 2\theta + C = 2\theta - 2\sin\theta\cos\theta + C$$

4.

$$\int e^{2x}\cos x\,dx = \frac{e^{2x}\sin x + 2e^{2x}\cos x}{5} + C$$

$u = e^{2x}$	$dv = \cos x\,dx$	$\tilde{u} = e^{2x}$	$d\tilde{v} = \sin x\,dx$
$du = 2e^{2x}dx$	$v = \sin x$	$d\tilde{u} = 2e^{2x}dx$	$\tilde{v} = -\cos x$

$$\int e^{2x}\cos x\,dx = e^{2x}\sin x - 2\int e^{2x}\sin x\,dx$$
$$= e^{2x}\sin x - 2\left[-e^{2x}\cos x - -2\int e^{2x}\cos x\,dx\right]$$
$$= e^{2x}\sin x + 2e^{2x}\cos x - 4\int e^{2x}\cos x\,dx$$
$$\Rightarrow\quad 5\int e^{2x}\cos x\,dx = e^{2x}\sin x + 2e^{2x}\cos x + \tilde{C}$$

5.

$$\int \frac{5x^3-3x^2+2x-1}{x^4+x^2}\,dx = 2\ln|x| + x^{-1} + \frac{3}{2}\ln|x^2+1| - 2\arctan x + C$$

$$\frac{5x^3-3x^2+2x-1}{x^2(x^2+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+1} = \frac{Ax(x^2+1)+B(x^2+1)+(Cx+D)(x^2)}{x^2(x^2+1)}$$

$$\Rightarrow\quad 5x^3-3x^2+2x-1 = Ax(x^2+1)+B(x^2+1)+(Cx+D)(x^2) \to x=0 \Rightarrow \boxed{-1=B}$$

$\Rightarrow$
- x^3 : $5 = A + C \Rightarrow \boxed{C=3}$
- x^2 : $-3 = B + D \Rightarrow \boxed{D=-2}$
- x : $\boxed{2 = A}$
- constant : $-1 = B$

$$\int \frac{5x^3-3x^2+2x-1}{x^4+x^2}\,dx = 2\int\frac{dx}{x} - 1\int x^{-2}\,dx + \frac{3}{2}\int\frac{2x\,dx}{x^2+1} - 2\int\frac{dx}{x^2+1}$$

6a. Let $n \in \mathbb{N} = \{1,2,3,4,5,\ldots\}$. Derive a reduction formula for the below integral.

$$\int (\ln x)^n\,dx = x(\ln x)^n - n\int(\ln x)^{n-1}\,dx$$

HINT: Your solution should look like:
$\int(\ln x)^n\,dx = $ *(some function of x)* + *(maybe a constant)* $\int(\ln x)^{\text{some number less than n}}\,dx$

$u = (\ln x)^n$	$dv = dx$
$du = n(\ln x)^{n-1}\cdot\frac{1}{x}\,dx$	$v = x$

$n=1$: $\int \ln x\,dx = x(\ln x)^1 - 1\int(\ln x)^0\,dx = x\ln x - \int dx$

$n=2$: $\int(\ln x)^2\,dx = x(\ln x)^2 - 2\int(\ln x)^1\,dx$
$= x(\ln x)^2 - 2(x\ln x - x) + C$

6b. Use your solution to **6a** to find the below 2 integrals.

$$\int \ln x\,dx = x\ln x - x + C$$

$$\int(\ln x)^2\,dx = x(\ln x)^2 - 2x\ln x + 2x + C$$

7. if you L'Hopital lots of times (specifically 10000 times) you get

$$\lim_{x\to\infty} \frac{x^{10000}}{e^x} = \lim_{x\to\infty} \frac{(\text{some constant})}{e^x} \underset{\uparrow\ \text{specifically}}{=} \lim_{x\to\infty} \frac{10000!}{e^x} = 0$$

8. $\lim_{x\to\infty} x^{\frac{1}{x}} = 1$

of form ∞^0 so:

$y = x^{1/x} \Rightarrow \ln y = \ln(x^{1/x}) = \frac{1}{x}\ln x$

$$\lim_{x\to\infty} \ln y = \lim_{x\to\infty} \frac{\ln x}{x} \underset{L'H}{\overset{\frac{\infty}{\infty}}{=}} \lim_{x\to\infty} \frac{\frac{1}{x}}{1} = \lim_{x\to\infty} \frac{1}{x} = 0$$

$$\ln y \xrightarrow{x\to\infty} 0 \Rightarrow y = e^{\ln y} \xrightarrow{x\to\infty} e^0 = 1$$

9. $\int_2^\infty \frac{\ln x\,dx}{x^2} = \frac{1+\ln 2}{2}$

$u = \ln x \quad dv = x^{-2}dx$ $du = \frac{dx}{x} \quad v = -x^{-1}$	$\int \frac{\ln x}{x^2}dx = -\frac{\ln x}{x} + \int x^{-2}dx = -\frac{\ln x}{x} - \frac{1}{x} + C.$

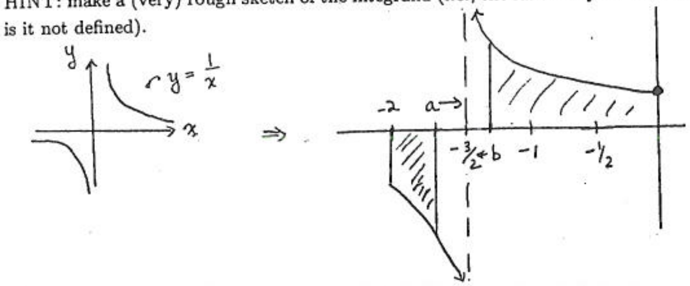
$$\int_2^\infty \frac{\ln x\,dx}{x^2} = \lim_{b\to\infty}\int_2^b \frac{\ln x\,dx}{x^2} = \lim_{b\to\infty}\left[-\frac{\ln x}{x} - \frac{1}{x}\Big|_{x=2}^{x=b}\right]$$

$$= \lim_{b\to\infty}\left[\frac{1+\ln x}{x}\Big|_{x=b}^{x=2}\right] = \lim_{b\to\infty}\left[\frac{1+\ln 2}{2} + \frac{1+\ln b}{b}\right]$$

$$= \frac{1+\ln 2}{2} + \lim_{b\to\infty}\frac{1+\ln b}{b} \underset{=}{\overset{L'H}{}} \frac{1+\ln 2}{2} + \boxed{\lim_{b\to\infty}\frac{1/b}{1}} \leftarrow = 0$$

10. $\int_{-2}^0 \frac{dx}{2x+3}$ = DNE or diverges.

HINT: make a (very) rough sketch of the integrand (i.e., the function you need to integrate - where is it not defined).



$$\int_{-2}^0 \frac{dx}{2x+3} = \lim_{a\to -3/2^-} \frac{1}{2}\int_{-2}^a \frac{2\,dx}{2x+3} + \lim_{b\to -3/2^+} \frac{1}{2}\int_b^0 \frac{2\,dx}{2x+3}$$

$$= \lim_{a\to -3/2^-} \frac{1}{2}\ln|2x+3|\Big|_{x=-2}^{x=a} + \lim_{b\to -3/2^+} \frac{1}{2}\ln|2x+3|\Big|_{x=b}^{x=0}$$

$$= \left[\lim_{a\to -3/2^-} \frac{\ln|2a+3|}{2}\right] - \frac{\ln 1}{2} + \frac{\ln 3}{2} - \lim_{b\to -3/2^+}\frac{\ln|2b+3|}{2}$$

$$= -\infty - 0 + \frac{\ln 3}{2} - -\infty$$