

$$\log_2(x+4) - \log_2 x = 3$$

for x . Your answer should not have a logarithm nor exponential in it.

ANSWER: $x = \frac{4}{7}$

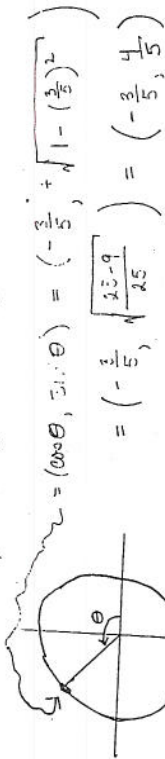
$$\log_2 \left(\frac{x+4}{x} \right) = 3 \Rightarrow \frac{x+4}{x} = 2^3 \Rightarrow 1 + \frac{4}{x} = 8 \Rightarrow \frac{4}{x} = 7$$

3. Simplify $\sin(2 \arccos(-\frac{3}{5}))$. Your answer should be a rational number.

ANSWER: $\sin(2 \arccos(-\frac{3}{5})) = 2(\frac{4}{5})(-\frac{3}{5}) = -\frac{24}{25}$

For this test I'll remind you that $\sin(2\theta) = 2 \sin \theta \cos \theta$ but you better know this by the next test!

Let $\theta = \arccos(-\frac{3}{5}) \Rightarrow \cos \theta = -\frac{3}{5}$



4a. $D_x [(3x+1)^2] = 2(3x+1)^1 \cdot 3 = 6(3x+1)$

4b. $D_x [(3x+1)^3] = (2^{3x+1}) (\ln 2) (3)$

4c. $D_x [(3x+1)^x] = (3x+1)^x \left[\ln(3x+1) + \frac{3x}{3x+1} \right]$

$y = (3x+1)^x \Rightarrow \ln y = x \ln(3x+1) \Rightarrow \frac{1}{y} \frac{dy}{dx} = \ln(3x+1) + x \frac{1}{3x+1} \quad (3)$

1. Let $f(x) = x^2 + 2x$ for $x \leq -1$.

1a. Find the inverse function of $y = f(x)$. Answer: $f^{-1}(x) =$

Let $y = f^{-1}(x)$. So $f(y) = x$ and $y \leq -1$. It follows that $y^2 + 2y = x$. I could use the quadratic formula. I will just complete the square.

$$(y+1)^2 = x+1$$

$$y+1 = \pm \sqrt{x+1}$$

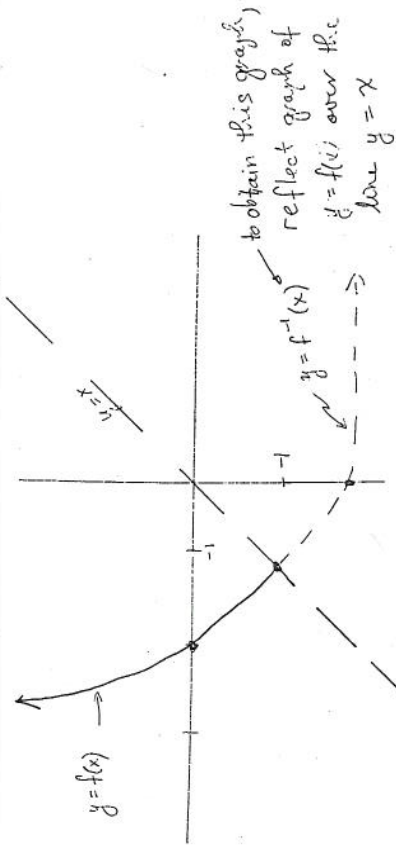
$$y = -1 \pm \sqrt{x+1}$$

But $y \leq -1$; so, $y = -1 - \sqrt{x+1}$. We conclude that

$$f^{-1}(x) = -1 - \sqrt{x+1}$$

1b. Sketch the graph of $y = f(x)$ and $y = f^{-1}(x)$ on the same grid. Explain how you got the graph of $y = f^{-1}(x)$ from the graph of $y = f(x)$.

$f(x) = x^2 + 2x = (x+1)^2 - 2 \Rightarrow x$ -intercepts at 0 and -2 \Rightarrow vertex at (-1, -2)



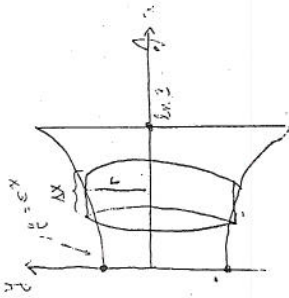
1c. The domain of $y = f^{-1}(x)$ is $x > -1$ or $[-1, \infty)$

8. The volume of the solid generated by revolving the region bounded by:
 $y = e^x$ and the x-axis and the y-axis and $x = \ln 3$

$$\frac{8\pi}{2} = 4\pi$$

about the x-axis is

Your answer should not have a logarithm nor exponential in it.



$$V_{\text{disk}} = (\text{area of base}) \cdot \text{height} = \pi \int_0^{\ln 3} [e^x]^2 dx$$

$$V = \pi \int_{x=0}^{x=\ln 3} [e^x]^2 dx = \pi \int_{x=0}^{x=\ln 3} e^{2x} dx$$

$$= \frac{\pi}{2} e^{2x} \Big|_{x=0}^{x=\ln 3} = \frac{\pi}{2} [e^{2 \ln 3} - e^0]$$

$$= \frac{\pi}{2} [e^{\ln 3^2} - 1] = \frac{\pi}{2} [9 - 1]$$

5.

$$D_x [3 \ln(1 + e^{5x})] = \frac{3 e^{5x} (5)}{1 + e^{5x}} = \frac{15 e^{5x}}{1 + e^{5x}}$$

6.

$$\int \frac{2 \ln x}{x} dx = 2 \int \ln x \left(\frac{dx}{x} \right) = 2 \int u du = u^2 + C = (\ln x)^2 + C$$

$$u = \ln x$$

$$du = \frac{dx}{x}$$

7.

$$\int e^{(3x+1)} dx = \frac{1}{3} \int e^u du = \frac{1}{3} e^u + C = \frac{1}{3} e^{3x+1} + C$$

$$u = 3x+1$$

$$du = 3dx$$

9. The solution to the differential equation

$$\frac{dy}{dt} = -6y(t)$$

$$y(t) = \frac{7e^{-6t}}{e^{-12}}$$

subject to the condition that $y(2) = 7$ is

You may assume that the $y(t) \geq 0$ for each t .

$$\frac{1}{y(t)} \frac{dy}{dt} = -6 \Rightarrow \int \frac{1}{y} dy = \int -6 dt \Rightarrow \ln |y| = -6t + C$$

$$\Rightarrow y(t) = e^{-6t+C} = e^C e^{-6t}$$

$$\Rightarrow y(2) = 7 = e^C e^{-12} \Rightarrow e^C = \frac{7}{e^{-12}}$$

$$\Rightarrow \frac{dP}{dt} = kP(t) \Rightarrow P(t) = P_0 e^{kt}$$

10. The rate of decay of a radioactive substance is proportional to the amount of such substance present.

Today we have 10 grams of a radioactive substance.

Given that one-third of the substance decays every 8 years, how much will be left t years from today? Clearly explain your notation.

$$\text{ANSWER: } P(t) = 10 \exp\left(\frac{t \ln\left(\frac{2}{3}\right)}{8}\right) \text{ grams}$$

Let t_0 = initial time = today.

t = the # of years after t_0

$P(t)$ = the # of grams of the radioactive substance at time t .

$$P(t) = 10 e^{kt}$$

$$P(8) = 10 e^{8k} = \frac{10}{3} \Rightarrow e^{8k} = \frac{2}{3} \Rightarrow 8k = \ln\left(\frac{2}{3}\right) \Rightarrow k = \frac{\ln\left(\frac{2}{3}\right)}{8}$$