

Complete the below 3 charts, similarly to the *Intro. to Taylor Polynomials* worksheet, which is posted (along with solutions) on the course homepage under **Selected Solutions**. Write your solutions so that the patterns for the c_n 's are easily recognizable (so leave factorials and constants raised to a power in your chart).

Example 1. Given the function $f(x) = \frac{1}{1-x}$ with center at $x_0 = 0$.

Helpful Table for Example 1			
n	$f^{(n)}(x)$	$f^{(n)}(x_0) \stackrel{\text{here}}{=} f^{(n)}(0)$	$c_n \stackrel{\text{def}}{=} \frac{f^{(n)}(x_0)}{n!} \stackrel{\text{here}}{=} \frac{f^{(n)}(0)}{n!}$
0	$(1-x)^{-1}$	$(1-0)^{-1} = 1$	$\frac{1}{0!} \stackrel{\text{note}}{=} \frac{0!}{0!} = 1$
1	$-(1-x)^{-2}(-1) = (1-x)^{-2}$	$(1-0)^{-2} = 1$	$\frac{1}{1!} = \frac{1!}{1!} = 1$
2	$-2(1-x)^{-3}(-1) = 2(1-x)^{-3}$	$2(1-0)^{-3} = 2$	$\frac{2}{2!} = \frac{2!}{2!} = 1$
3	$2(-3)(1-x)^{-4}(-1) = 3!(1-x)^{-4}$	$3!(1-0)^{-4} = 3!$	$\frac{3!}{3!} = 1$
4	$3!(-4)(1-x)^{-5}(-1) = 4!(1-x)^{-5}$	$4!(1-0)^{-5} = 4!$	$\frac{4!}{4!} = 1$
5	$4!(-5)(1-x)^{-6}(-1) = 5!(1-x)^{-6}$	$5!(1-0)^{-6} = 5!$	$\frac{5!}{5!} = 1$
6	$5!(-6)(1-x)^{-7}(-1) = 6!(1-x)^{-7}$	$6!(1-0)^{-7} = 6!$	$\frac{6!}{6!} = 1$

Example 2. Given the function $f(x) = \sin x$ with center at $x_0 = \pi$.

Helpful Table for Example 2			
n	$f^{(n)}(x)$	$f^{(n)}(x_0) \stackrel{\text{here}}{=} f^{(n)}(\pi)$	$c_n \stackrel{\text{def}}{=} \frac{f^{(n)}(x_0)}{n!} \stackrel{\text{here}}{=} \frac{f^{(n)}(\pi)}{n!}$
0	$\sin x$	$\sin \pi = 0$	$\frac{0}{0!} = \frac{0}{1} = 0$
1	$\cos x$	$\cos \pi = -1$	$\frac{-1}{1!} = -\frac{1}{1} = -1$
2	$-\sin x$	$-\sin \pi = 0$	$\frac{0}{2!} = 0$
3	$-\cos x$	$-\cos \pi = -(-1) = 1$	$\frac{1}{3!} = +\frac{1}{3!}$
4	$\sin x$	$\sin \pi = 0$	$\frac{0}{4!} = 0$

► Note $f^{(4)}(x) = f^{(0)}(x)$ so the derivatives $y = f^{(n)}(x)$ repeat/cycle in sets of 4.

Example 3. Given the function $f(x) = \ln(1+x)$ with center $x_0 = 0$.

Helpful Table for Example 3			
n	$f^{(n)}(x)$	$f^{(n)}(x_0) \stackrel{\text{here}}{=} f^{(n)}(0)$	$c_n \stackrel{\text{def}}{=} \frac{f^{(n)}(x_0)}{n!} \stackrel{\text{here}}{=} \frac{f^{(n)}(0)}{n!}$
0	$\ln(1+x)$	$\ln(1+0) = 0$	$\frac{0}{0!} = \frac{0}{1} = 0$
1	$(1+x)^{-1}$	$(1+0)^{-1} = +1$	$\frac{1}{1!} = +1$
2	$-(1+x)^{-2}$	$-(1+0)^{-2} = -1$	$\frac{-1}{2!} = -\frac{1}{2}$
3	$+2(1+x)^{-3}$	$+2(1+0)^{-3} = +2$	$\frac{2}{3!} = +\frac{1}{3}$
4	$-3!(1+x)^{-4}$	$-3!(1+0)^{-4} = -3!$	$\frac{-3!}{4!} = -\frac{1}{4}$
5	$+4!(1+x)^{-5}$	$+4!(1+0)^{-5} = +4!$	$\frac{4!}{5!} = +\frac{1}{5}$
6	$-5!(1+x)^{-6}$	$-5!(1+0)^{-6} = -5!$	$\frac{-5!}{6!} = -\frac{1}{6}$