PIN: **NAME:** **Due: Tuesday Feb. 5, 2019**

In this assignment you are given a function $y = f(z)$, where $f\colon (0,\infty) \to \mathbb{R}$, and you are asked to find the limit of $y = f(z)$ as $z \to \infty$, i.e.,

$$\lim_{z\to\infty} f(z) \qquad \text{which can also be written as} \qquad \lim_{\substack{z\to\infty\\ z\in\mathbb{R}}} f(z)\,.$$

The limits in this assignment are used often in Calc. II and can be done using just Calc. I knowledge. Recall: the handout *Indetermine Forms (L'Hôpital's Rule)*, which is a summary/review of L'H rule, can be found on the course homepage under Handouts (see **Preparation**).

Instructions.
First compute the limits by-hand, showing all your work below the box and then putting your answer in the box. **Next** use Maple to check your answers. **Hand in** only your by-hand work.

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1. $\displaystyle\lim_{\substack{z\to\infty\\ z\in\mathbb{R}}} \frac{\ln z}{z} = \boxed{}$ Hint: L'Hôpital's Rule.

2. $\displaystyle\lim_{\substack{z\to\infty\\ z\in\mathbb{R}}} z^{1/z} = \boxed{}$

Hint: $\displaystyle\lim_{z\to\infty} z^{1/z}$ has the indetermine form ∞^0 so consider $\ln\left(z^{1/z}\right) \overset{\text{i.e.}}{=} \left(\frac{1}{z}\right)\ln z \overset{\text{i.e.}}{=} \frac{\ln z}{z}$, and then apply L'H.

3. Let $c > 0$ be a positive constant. $\displaystyle\lim_{\substack{z\to\infty\\ z\in\mathbb{R}}} c^{1/z} = \boxed{}$

Hint: $\ln\left(c^{1/z}\right) = \left(\frac{1}{z}\right)\ln c = \frac{\ln c}{z}$ and since c is a constant, $\ln c$ is also a constant.

4. Let c be a constant. Think of c as a fixed number, for example $\frac{1}{17}$.

4a. If $0 < c < 1$ then $\lim\limits_{\substack{z\to\infty \\ z\in\mathbb{R}}} c^z = \boxed{}$

Hint: $\ln(c^z) = z \ln c$ and since $0 < c < 1$ is a constant, $\ln c$ is also a constant and $\ln c < 0$.

4b. If $c = 1$ then $\lim\limits_{\substack{z\to\infty \\ z\in\mathbb{R}}} c^z = \boxed{}$ Hint: For each $z \in \mathbb{R}$ we have $c^z = 1^z = 1$.

4c. If $c > 1$ then $\lim\limits_{\substack{z\to\infty \\ z\in\mathbb{R}}} c^z = \boxed{}$

Hint: $\ln(c^z) = z \ln c$ and since $c > 1$ is a constant, $\ln c$ is also a constant and $\ln c > 0$.

5. Let c be a constant. $\lim\limits_{\substack{z\to\infty \\ z\in\mathbb{R}}} \left(1 + \frac{c}{z}\right)^z = \boxed{}$

Hint: $\lim\limits_{z\to\infty} \left(1 + \frac{c}{z}\right)^z$ has the indetermine form 1^∞ so consider $\ln\left(\left(1 + \frac{c}{z}\right)^z\right)$ and rewrite $\ln\left(\left(1 + \frac{c}{z}\right)^z\right) = z \ln\left(1 + \frac{c}{z}\right) = \frac{\ln\left(1 + \frac{c}{z}\right)}{\frac{1}{z}} \overset{\text{also}}{=} \frac{\ln\left(\frac{z+c}{z}\right)}{\frac{1}{z}}$. Now apply L'H. If needed, use&connect another sheet of paper.