

This quiz, which you can do after §8.2, is **DUE Tuesday February 5** at the **beginning** of class. **First**, read pages 504–509 from the Thomas textbook (covering Improper Integral); specifically, read from the start of §8.8 through the subsection *Improper Integrals with a CAS*. Recall below details of FTC.

The Fundamental Theorem of Calculus (FTC) gives that if  $f$  is continuous on  $[a, b]$ , then

$$\int_a^b f(x) dx = F(b) - F(a)$$

where  $F$  is any antiderivative of  $f$ , i.e., a function such that  $F' = f$ . Note that to apply the FTC, one needs  $f$  to be continuous on the whole interval  $[a, b]$ . Thus one needs the following 3 conditions.

(1) If  $a < c < b$  (in other words, if  $c \in (a, b)$ ), then

$$\lim_{x \rightarrow c} f(x) = f(c) .$$

(2) The limit of  $y = f(x)$  as  $x$  approaches  $a$  from the right is  $f(a)$ , which can be expressed

$$\lim_{x \rightarrow a^+} f(x) = f(a) .$$

(3) The limit of  $y = f(x)$  as  $x$  approaches  $b$  from the left is  $f(b)$ , which can be expressed

$$\lim_{x \rightarrow b^-} f(x) = f(b) .$$

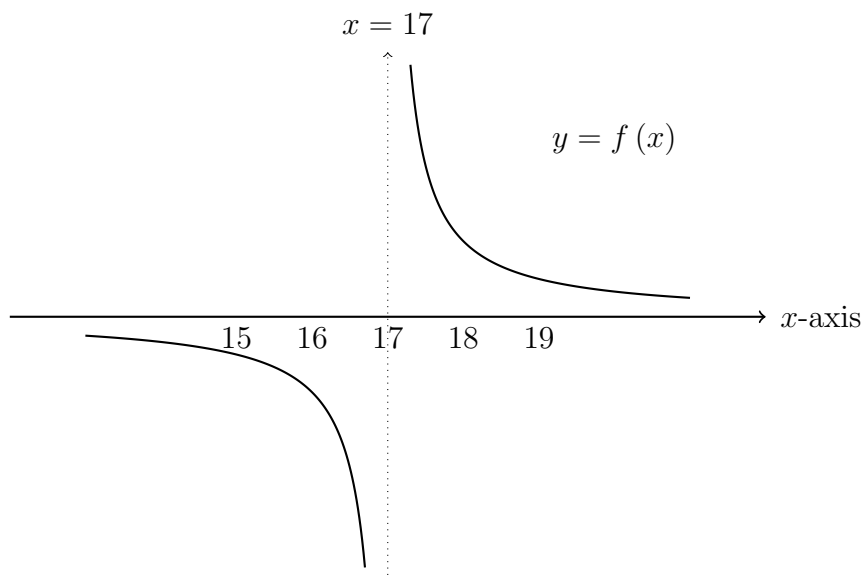
Consider the function

$$f(x) = \frac{1}{(x-17)^3} .$$

Note that the function  $y = f(x)$  is not defined at  $x = 17$  and is continuous on  $\mathbb{R} \setminus \{17\}$ .

Recall  $\mathbb{R} \setminus \{17\}$  denotes the whole real line *take away* the point 17, so  $\mathbb{R} \setminus \{17\} = (-\infty, 17) \cup (17, \infty)$ .

Below is a rough sketch of the graph of  $y = f(x)$ .



**Next** answer the questions on the next page.

Answer the following questions.

1. Fill in the 3 boxes as so to express the following integral as an appropriate limit of definite integrals  $\int_a^b f(x) dx$  where  $y = f(x)$  is continuous on  $[a, b]$ .

$$\int_{18}^{\infty} \frac{1}{(x-17)^3} dx = \lim_{\boxed{\phantom{000}}} \int_{x=\boxed{\phantom{000}}}^{x=\boxed{\phantom{000}}} f(x) dx .$$

2. Fill in the 3 boxes as so to express the following integral as an appropriate limit of definite integrals  $\int_a^b f(x) dx$  where  $y = f(x)$  is continuous on  $[a, b]$ .

$$\int_{17}^{18} \frac{1}{(x-17)^3} dx = \lim_{\boxed{\phantom{000}}} \int_{x=\boxed{\phantom{000}}}^{x=\boxed{\phantom{000}}} f(x) dx .$$

3. Fill in the 3 boxes as so to express the following integral as an appropriate limit of definite integrals  $\int_a^b f(x) dx$  where  $y = f(x)$  is continuous on  $[a, b]$ .

$$\int_{16}^{17} \frac{1}{(x-17)^3} dx = \lim_{\boxed{\phantom{000}}} \int_{x=\boxed{\phantom{000}}}^{x=\boxed{\phantom{000}}} f(x) dx .$$

4. Fill in the 6 boxes as so to express the following integral as a sum of two limits of definite integrals  $\int_a^b f(x) dx$  where  $y = f(x)$  is continuous on  $[a, b]$ .

$$\int_{16}^{18} \frac{1}{(x-17)^3} dx = \lim_{\boxed{\phantom{000}}} \int_{x=\boxed{\phantom{000}}}^{x=\boxed{\phantom{000}}} f(x) dx + \lim_{\boxed{\phantom{000}}} \int_{x=\boxed{\phantom{000}}}^{x=\boxed{\phantom{000}}} f(x) dx .$$

5. Express the following integral as a sum of appropriate limits of definite integrals  $\int_a^b f(x) dx$  where  $y = f(x)$  is continuous on  $[a, b]$ .

$$\int_{16}^{\infty} \frac{1}{(x-17)^3} dx =$$