1. Use either the *Comparison Test* or *Limit Comparison Test* to determine whether the series converges or diverges, and *justify your answer.*

$$\sum_{n=100}^{\infty} \frac{n}{n^3 - 2}$$

☒ conv

☐ divg

Here, $a_n = \frac{n}{n^3-2}$

• $0 \le a_n \overset{\text{given}}{=} \frac{n}{n^3-2} \overset{n\text{ big}}{\approx} \frac{n}{n^3} = \frac{1}{n^2} \overset{\text{let}}{=} b_n \Rightarrow$ compare $\sum \frac{n}{n^3-2}$ to $\sum \frac{1}{n^2}$,

• $\sum b_n = \sum \frac{1}{n^2}$ conv. (b/c p-series, $p = 2 > 1$), so make Ansatz $\sum a_n$ conv.

WAY #1 LCT

• $\lim\limits_{n\to\infty} \frac{a_n}{b_n} = \lim\limits_{n\to\infty} \left(\frac{n}{n^3-2}\right)\left(\frac{n^2}{1}\right) \overset{A}{=} \lim\limits_{n\to\infty} \frac{1n^3}{1n^3-2} \overset{A}{=} \lim\limits_{n\to\infty} \frac{1}{1-\frac{2}{n^3}} = 1$ and $0 < 1 < \infty$,

• $\sum \frac{1}{n^2}$ Conv (b/c p-series, $p = 2 > 1$)

• So LCT ~~says~~ says $\sum \frac{n}{n^3-2}$ conv.

WAY #2 CT. To show $\sum a_n$ conv., we need to ↓ bound above by a convergent ↓ series $\sum c_n$ < we think $c_n \sim \frac{1}{n^2}$ >

2. Use the *Ratio Test* to determine if the series converges or diverges, and *justify your answer.*

$$\sum_{n=1}^{\infty} \frac{(n-1)!}{(n+1)^2}$$

☐ conv

☒ divg

$$\frac{a_{n+1}}{a_n} = \frac{([n+1]-1)!}{([n+1]+1)^2} \cdot \frac{(n+1)^2}{(n-1)!} \overset{A}{=} \frac{n!}{(n-1)!} \frac{(n+1)^2}{(n+2)^2}$$

$$\overset{A}{=} \frac{(n-1)!\, n}{(n-1)!} \left(\frac{n+1}{n+2}\right)^2 \overset{A}{=} n \left(\frac{1 n+1}{1n+2}\right) \xrightarrow{n\to\infty} \infty \cdot \left(\frac{1}{1}\right) = \infty,$$

Since $\lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \infty > 1$, $\sum a_n$ divg.

continued.

• $0 \le a_n = \frac{n}{n^3-2} \overset{\text{for } n \text{ big}}{\le} \frac{n}{n^3 - \frac{1}{17}n^3} \overset{A}{=} \frac{n}{\frac{16}{17}n^3} = \frac{17}{16}\frac{1}{n^2} \overset{\text{let}}{=} c_n$

$\Leftrightarrow n^3 - 2 \ge n^3 - \frac{1}{17}n^3 \Leftrightarrow 2 \le \frac{1}{17}n^3 \Leftrightarrow \sqrt[3]{2(17)} \le n.$

• $\sum \frac{17}{16}\frac{1}{n^2} = \frac{17}{16}\sum \frac{1}{n^2}$ conv (p-series, $p = 2 > 1$).

• So the CT gives $\sum \frac{n}{n^3-2}$ conv.