Sequences

(with some Maple Commands/Examples needed for Series to come)

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Overview

Sequences and series are the objects of interest for the next few weeks. The intent of this lab is to provide additional practice determining the convergence or divergence of a sequence of numbers. Ways to generate sequences and series in Maple are also introduced.

Maple Essentials

- New Maple commands introduced in this lab:

Command/Example	Description
Sequence Part - For this Lab	

`evalf(`*expression*`);`
Example:
`evalf(Pi);`
	numerically evaluates expressions involving constants

`seq(f(n), n=i..j);`
Example:
`seq(1/n,n=1..10);`
	creates a finite sequence of values
	$f(i)$, $f(i+1)$, $\cdots$ $f(j)$,
	where $f(n)$ is a maple function and $i \leq j$ are integers.

`seq([n,f(n)],n=i..j);`
Example:
`f:=x->x^2;`
`seq([n,f(n)],n=1..10);`
	creates a finite sequence of points on the graph of $y = f(x)$.

Series Part - For Labs To Come	

`sum(f(n), n=i..j)`
Example:
`sum(n^2, n=1..10);`
	creates and evaluates a finite or infinite sum, that is, series $\sum_{n=i}^{j} f(n)$, where `f(n)` is a maple function or expression and $i \leq j$ can be integers, variables, or infinity. For a finite or convergent infinite series, it automatically evaluates the sum and returns a value or formula. If you don't want the automatic evaluation, use `Sum` instead of `sum`.

`for n from i to j do...end do;`
Example:
`s[1]:=1; for n from 1 to 9 do`
`s[n+1]:= s[n]+n end do;`
	A typical for-loop (`for` and `do` statement) used in general programming languages. It executes whatever between ``` ``do'' ``` and ``` ``end do'' ``` repeatedly for a counted number of times (``` ``for n from i to j'' ```). It hence can be used to work with sequences in much more general ways than what the command `seq` could.

- A link to the *SequenceDrill* maplet can be found on the course website:

`http://www.math.sc.edu/~girardi/m142/11F/maple/maple.html` → SequenceDrill

Preparation

Sections 11.1 in Stewart. Sections 9.1 in CalcLabs. In addition, review the basic qualitative properties of logarithms, powers, exponentials, and so on. For example, exponentials grow faster (at ∞) than polynomials, factorials grow faster than exponentials, and so on.

Activities

For the given sequence $\{a_n\}_{n=1}^{\infty}$, do the following.

(a) Generate the first 10 terms. I.e., find $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9$, and a_{10}.

(b) Graph enough points (n, a_n) to make a guess as to whether the the sequence $\{a_n\}_{n=1}^{\infty}$ converges (in which case, what is $\lim_{n\to\infty} a_n$?) or diverges.

(c) See if your guess in part (b) agrees with what Maple thinks the $\lim_{n\to\infty} a_n$ is.

(1) $a_n = \arctan(n)$ (2) $a_n = \cos(n\pi)$ (3) $a_n = \dfrac{(-1)^n}{n}$

Note: *You may use the* SequenceDrill *maplet.*

Example: Activity 1a,b,c.

```
> with(plots):
> f:= n-> arctan(n);
> evalf(seq(f(n), n=1..10));
> points:=evalf(seq([n,f(n)], n=1..10));
> P1:=plot([points], style=point):
> P2:=plot([-1/2*Pi, 1/2*Pi]):
> display([P1,P2]);
> limit(f(n), n=infinity);
```

Assignment: **due Wed 9/21 at BEGINNING of recitation**

For the given sequence $\{a_n\}_{n=1}^{\infty}$, do the following.

(a) Generate the first 10 terms. I.e., find $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9$, and a_{10}.

(b) Graph enough points (n, a_n) to make a guess as to whether the the sequence $\{a_n\}_{n=1}^{\infty}$ converges (in which case, what is $\lim_{n\to\infty} a_n$?) or diverges.

(c) **Verify** your guess in part (b) by hand using the Limit Laws and Theorems about sequences from Section 11.1 of our textbook.

(1) $\{1-(0.2)^n\}_{n=1}^{\infty}$ (2) $\left\{\dfrac{3+5n^2}{n+n^2}\right\}_{n=1}^{\infty}$ (3) $\left\{e^{1/n}\right\}_{n=1}^{\infty}$

(4) $\left\{\dfrac{2n\pi}{1+8n}\right\}_{n=1}^{\infty}$ (5) $\left\{\left(1+\dfrac{2}{n}\right)^n\right\}_{n=1}^{\infty}$ (6) $\left\{\dfrac{n!}{2^n}\right\}_{n=1}^{\infty}$