NEWTON POLYGONS

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WITH RESPECT TO A PRIME p

$$f(x) \quad = \quad x^6 + \ 24\ x^5 + \ 12\ x^3 - \ 18\ x + \ 36$$

$$f(x) \quad = \quad x^6 + 2^3 3^1 x^5 + \ 12\ x^3 - \ 18\ x + \ 36$$

$$f(x) \quad = \quad x^6 + 2^3 3^1 x^5 + 2^2 3^1 x^3 - \ 18\ x + \ 36$$

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$$f(x) = 2^0 3^0 x^6 + 2^3 3^1 x^5 + 2^2 3^1 x^3 - 2^1 3^2 x + 2^2 3^2$$

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$$p = 2$$

$$f(x) = 2^{0}3^{0}x^{6} + 2^{3}3^{1}x^{5} + 2^{2}3^{1}x^{3} - 2^{1}3^{2}x + 2^{2}3^{2}$$

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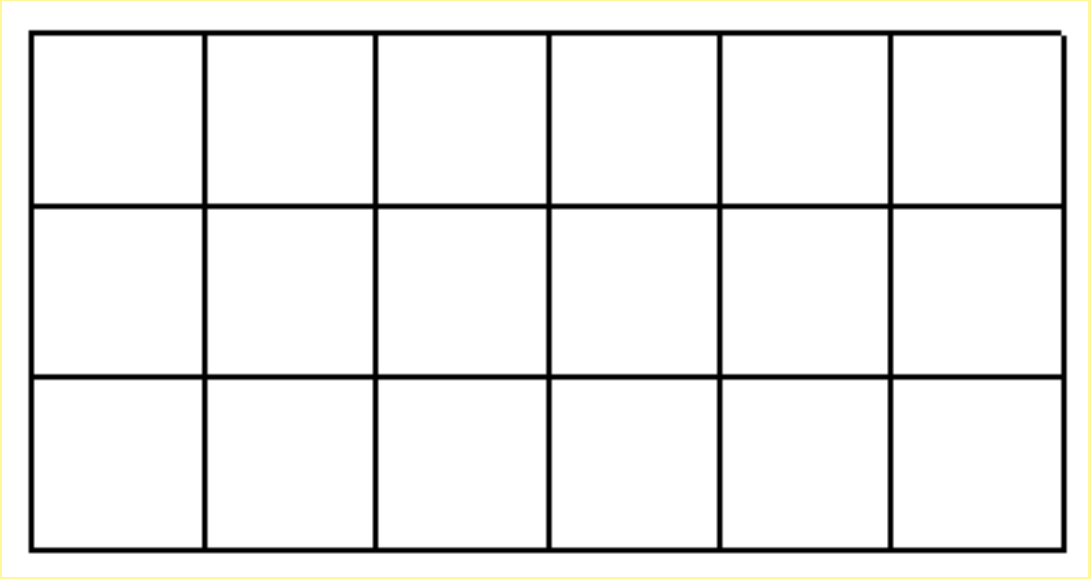
$$p = 2$$



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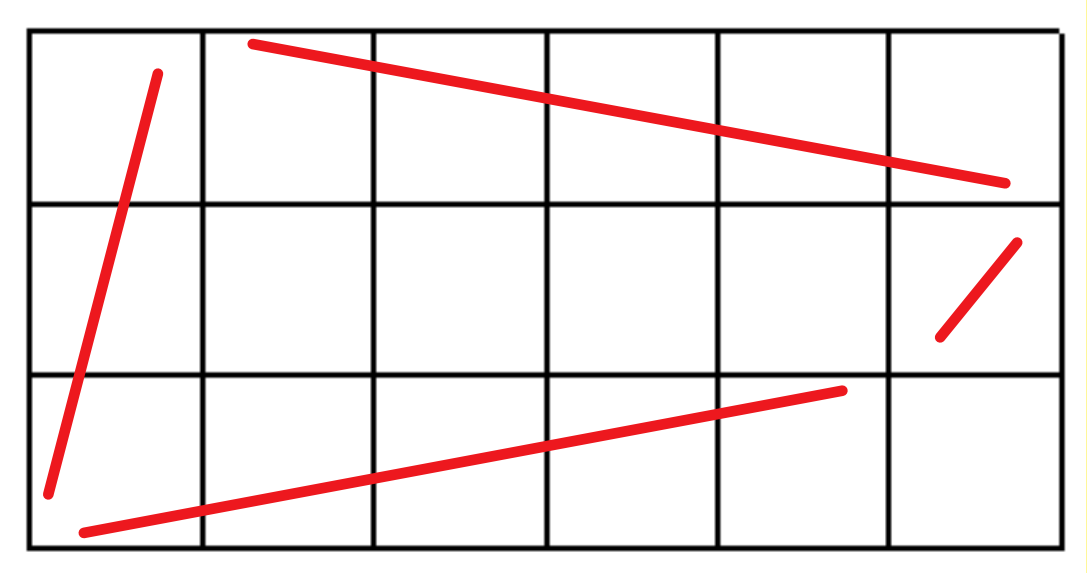

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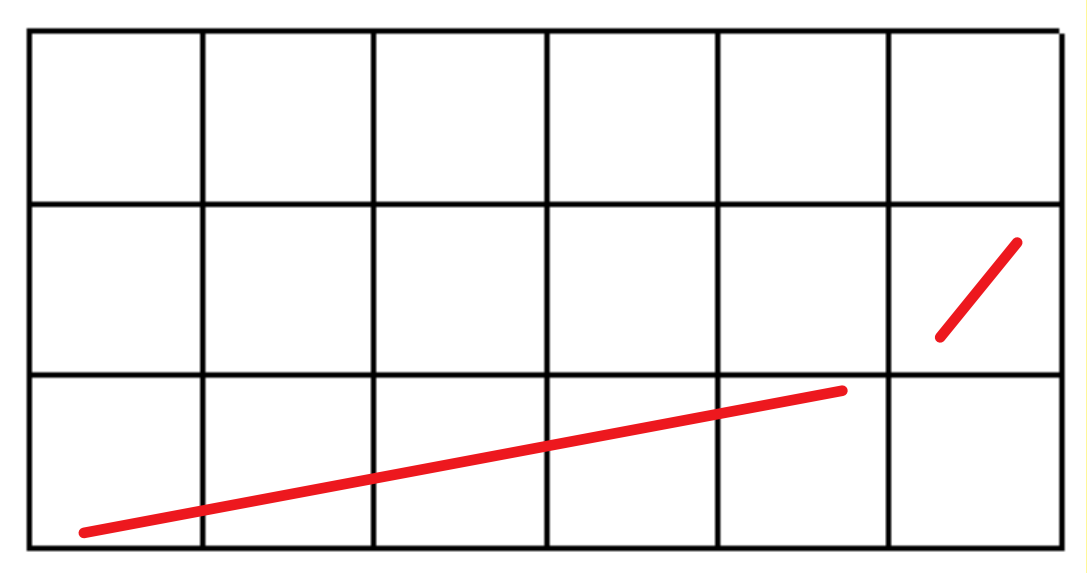

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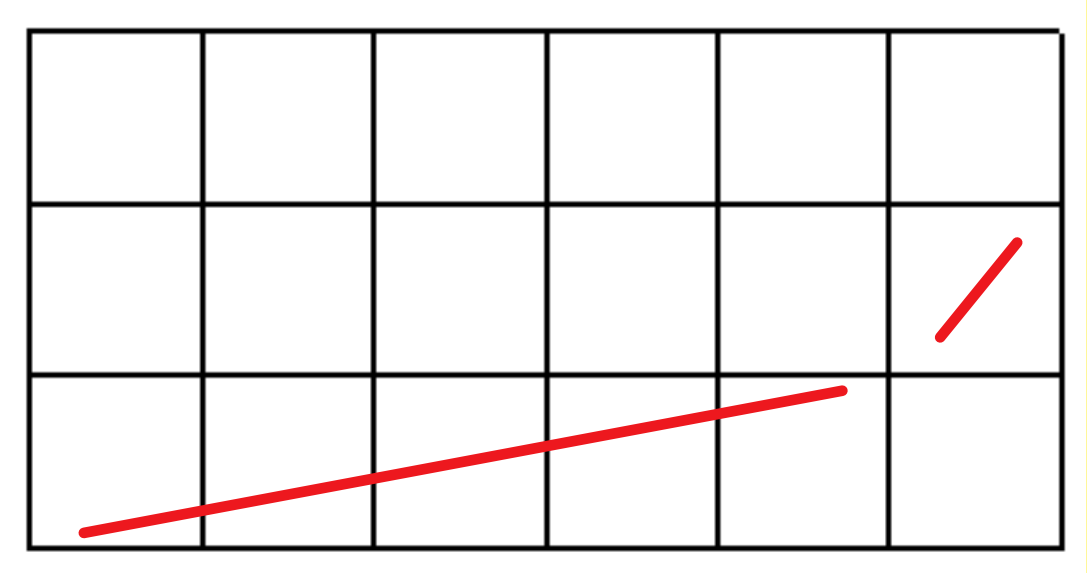
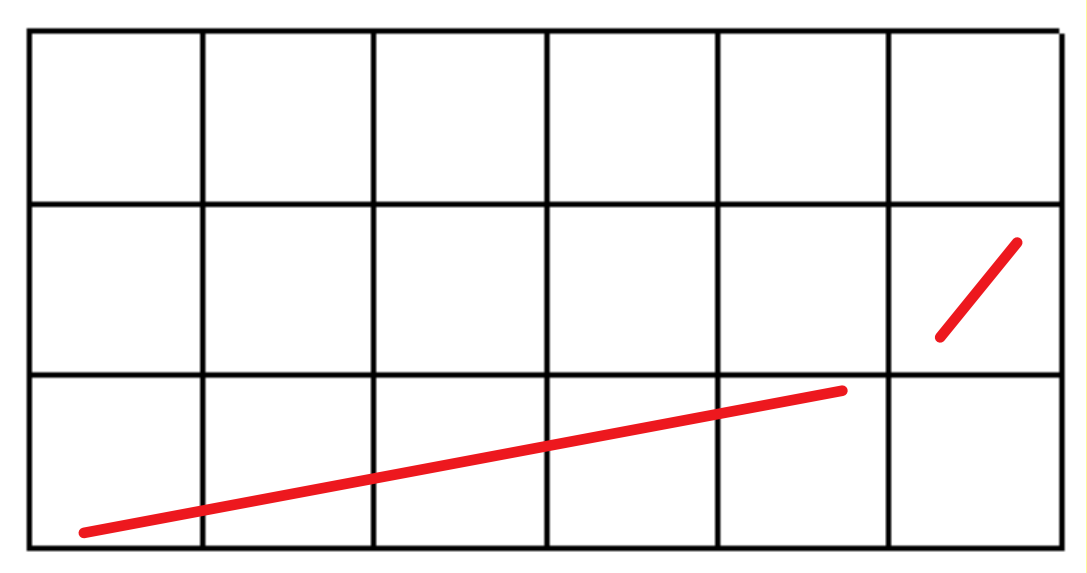


Newton polygon of $f(x)$ with respect to p

$$f(x) = 2^{0}3^{0}x^{6} + 2^{3}3^{1}x^{5} + 2^{2}3^{1}x^{3} - 2^{1}3^{2}x + 2^{2}3^{2}$$

$$p = 2$$



Newton polygon of $f(x)$ with respect to 2

Comment: The slopes of the edges of a Newton polygon increase going from left to right.

$$f(x) = 2^0 3^0 x^6 + 2^3 3^1 x^5 + 2^2 3^1 x^3 - 2^1 3^2 x + 2^2 3^2$$

$$f(x) = 2^0 3^0 x^6 + 2^3 3^1 x^5 + 2^2 3^1 x^3 - 2^1 3^2 x + 2^2 3^2$$

$$p = 3$$

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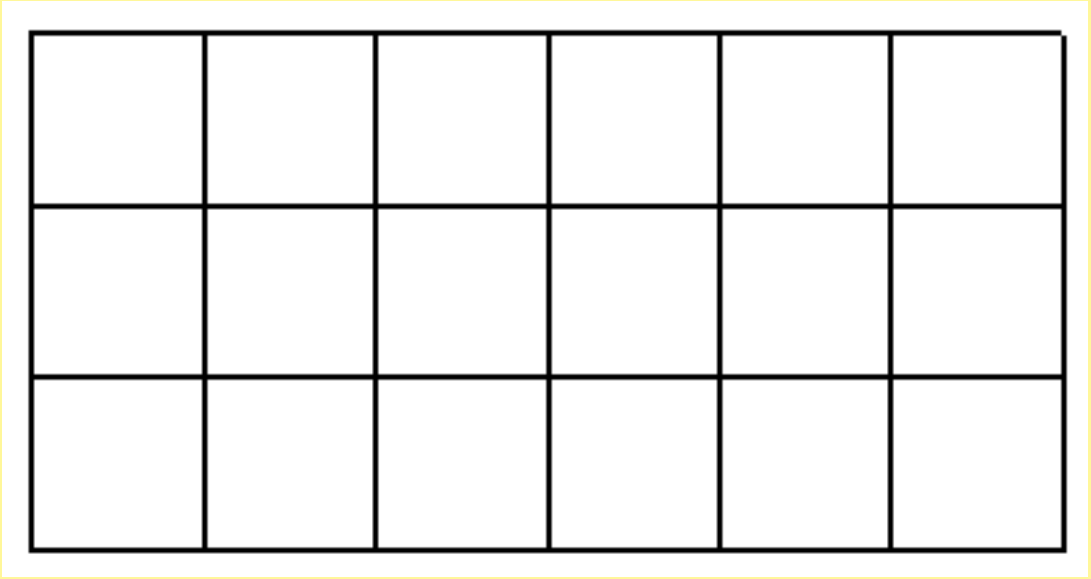

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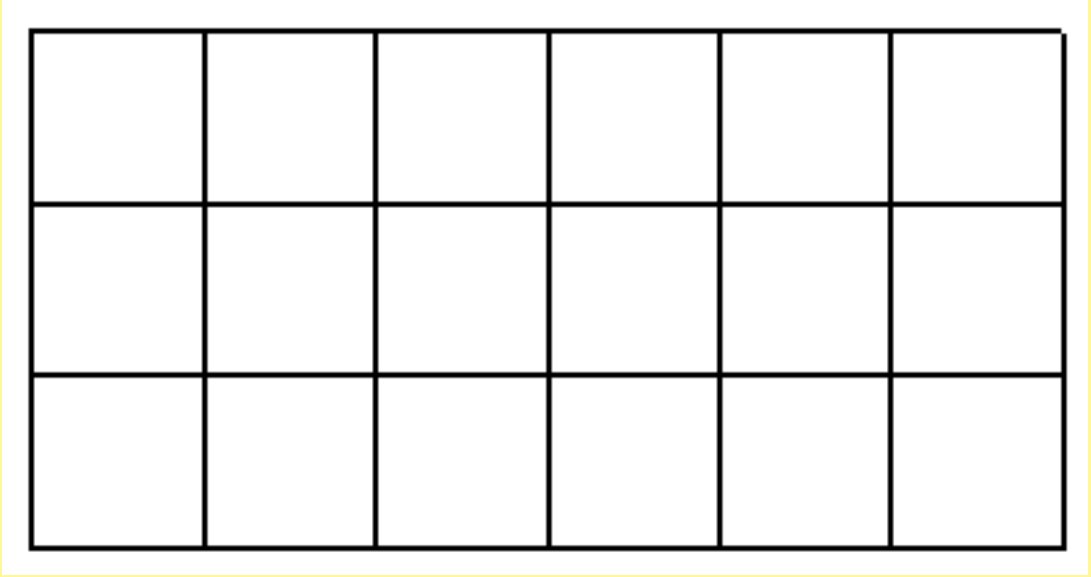

0

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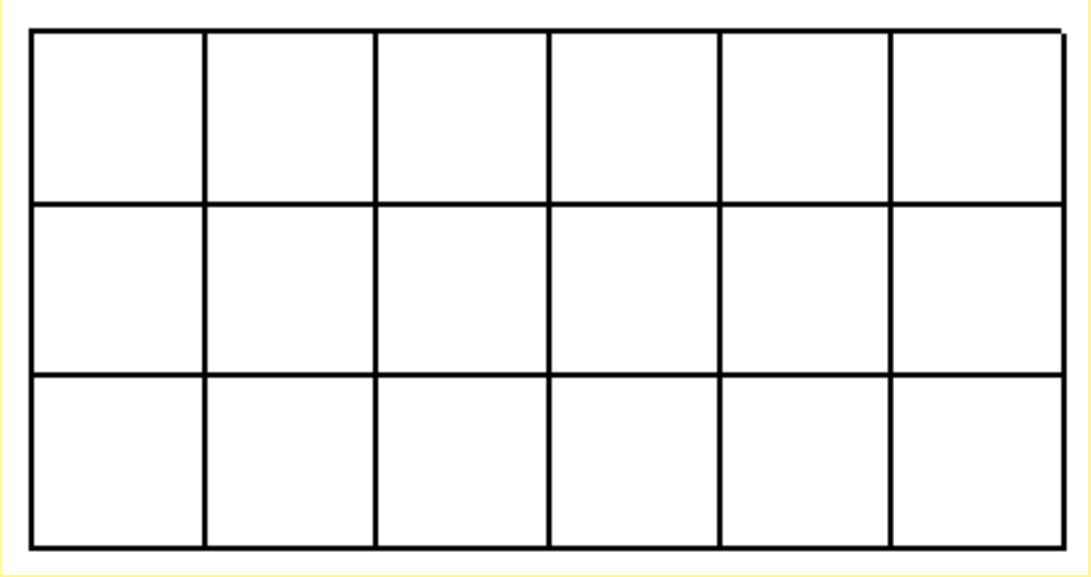
$$p = 3$$



0 1

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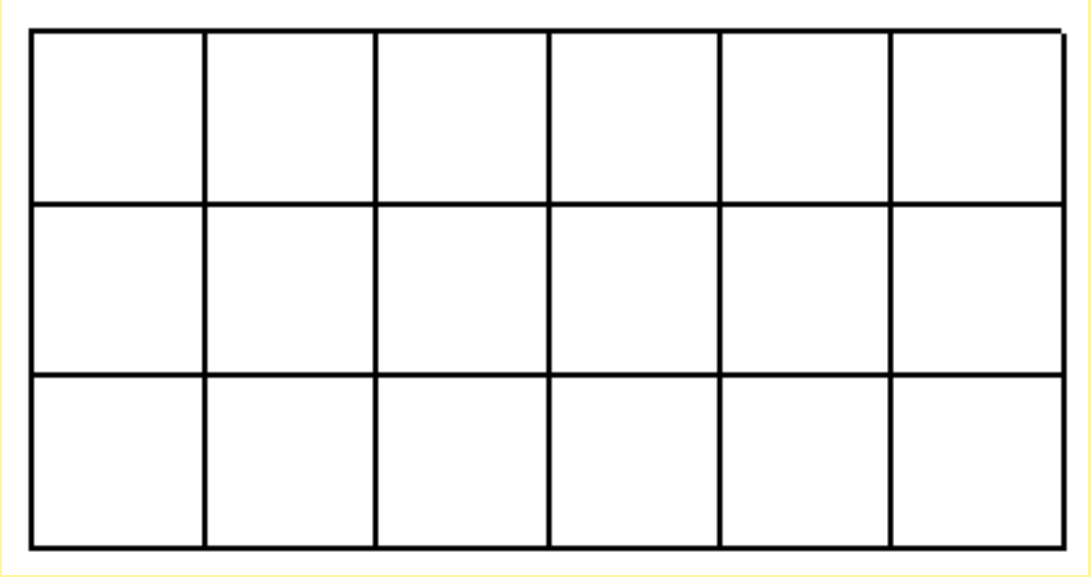
$$p = 3$$



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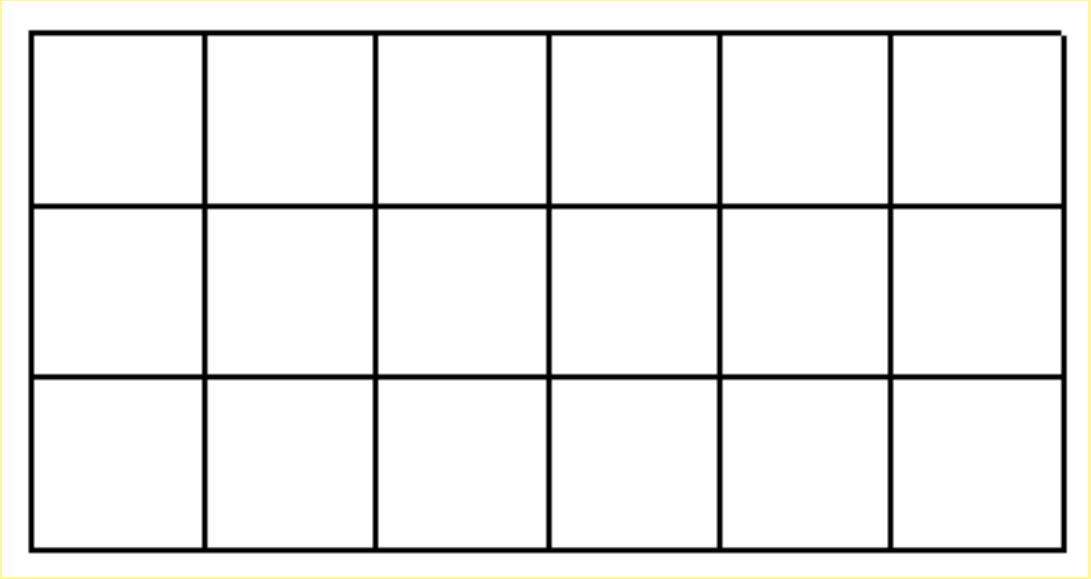

0 1 1 2

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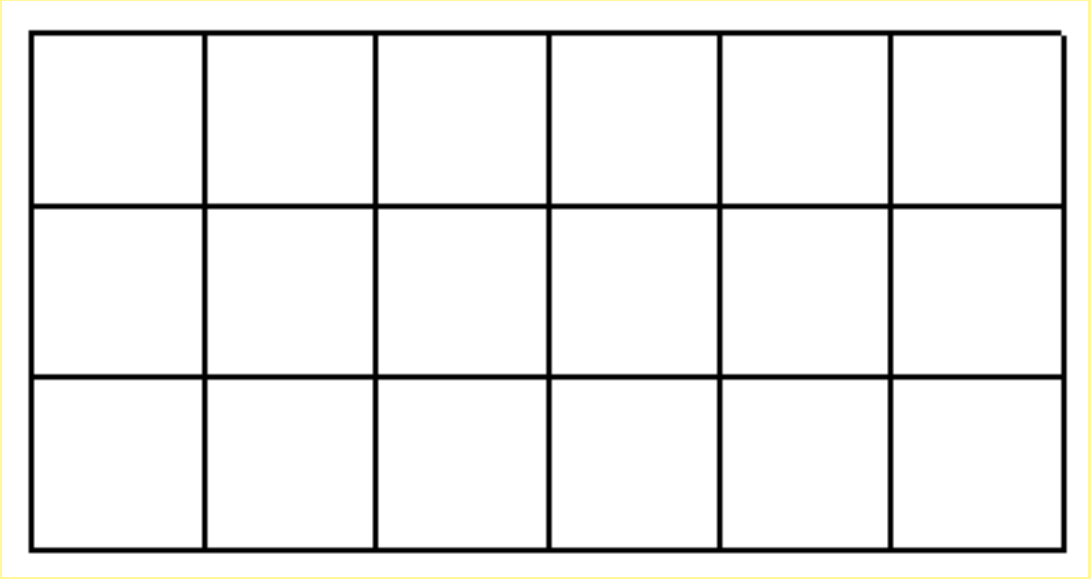

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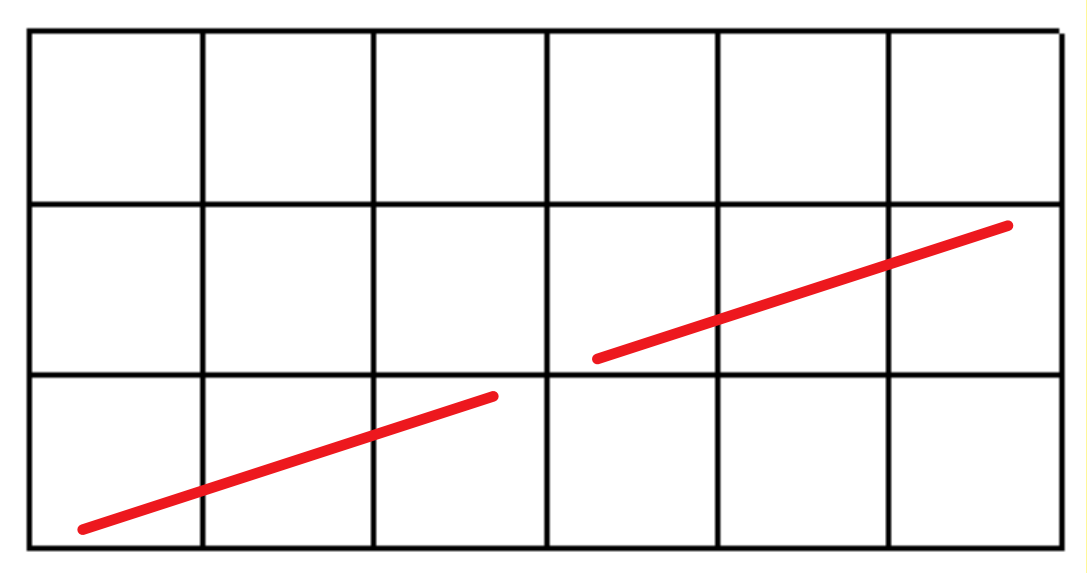


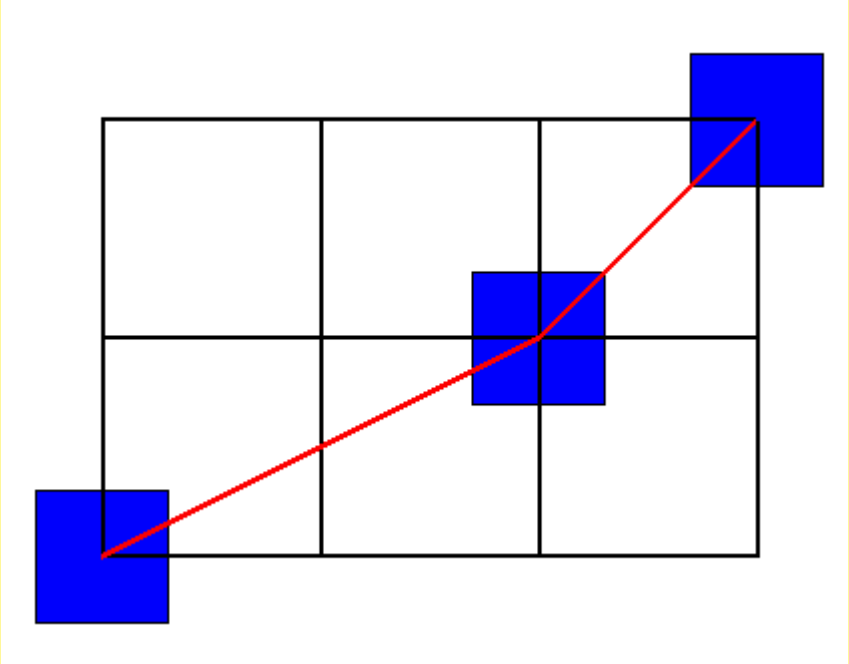
Newton polygon of $f(x)$ with respect to 3

DUMAS' THEOREM

DUMAS' THEOREM . . .

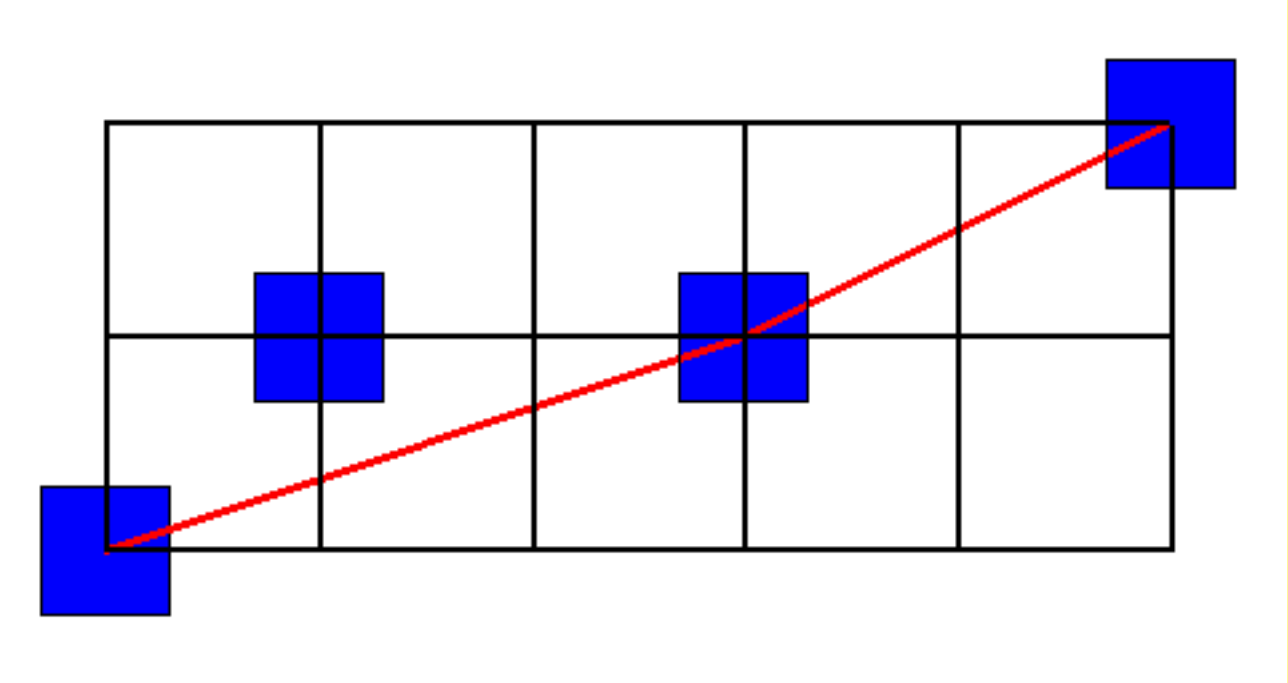
$$g(x) = x^3 + 2x - 4, \;\; h(x) = x^5 - 6x^4 + 2x^2 - 12$$

$$g(x) = x^3 + 2x - 4, \quad h(x) = x^5 - 6x^4 + 2x^2 - 12$$

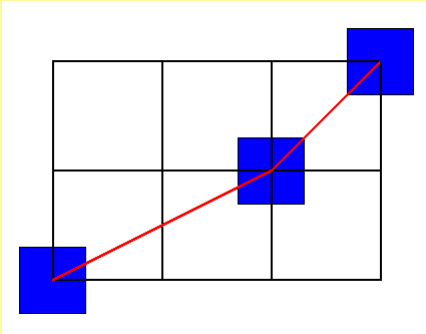


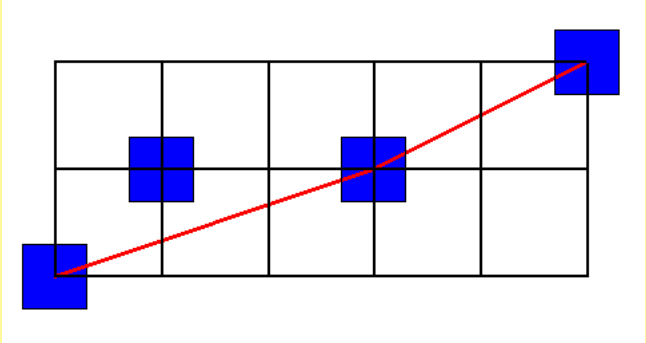
Newton polygon of $g(x)$ with respect to 2

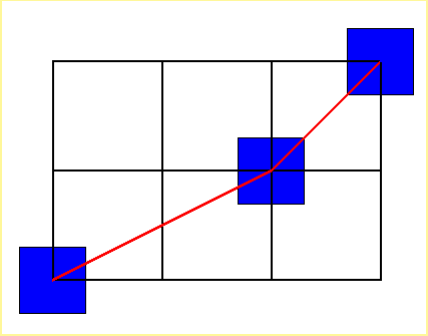
$$g(x) = x^3 + 2x - 4, \;\; h(x) = x^5 - 6x^4 + 2x^2 - 12$$

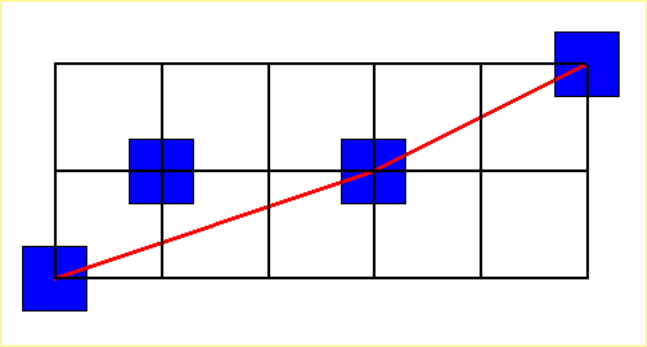


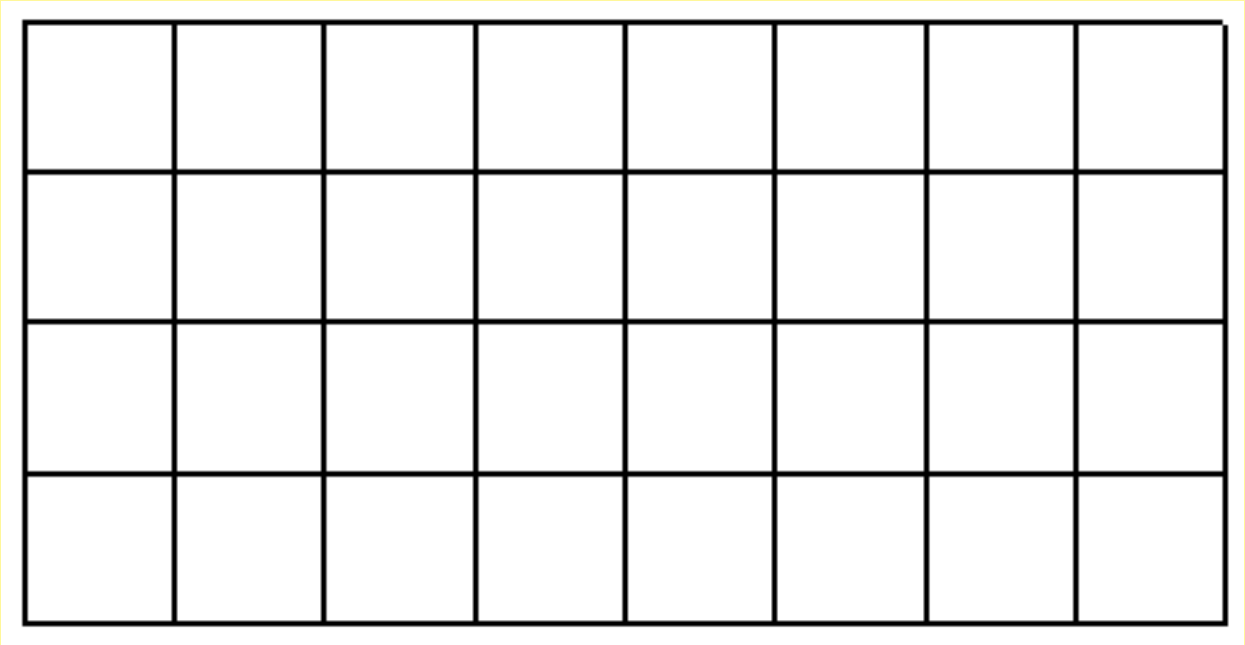
Newton polygon of $h(x)$ with respect to 2



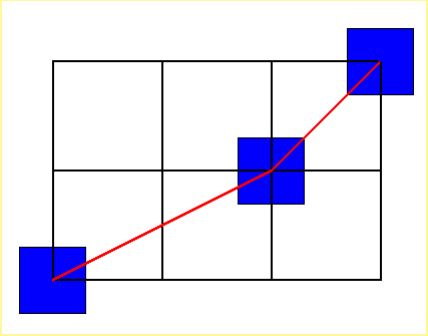


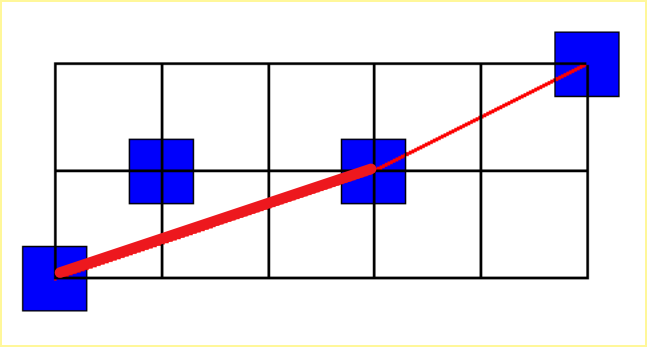


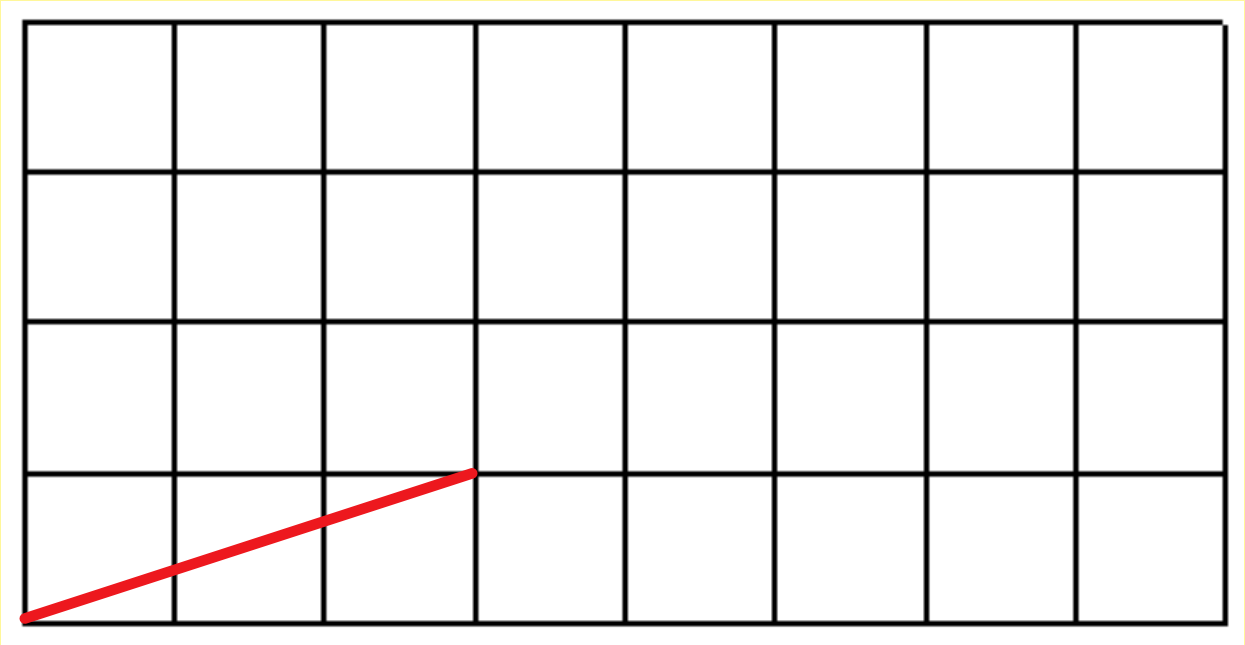




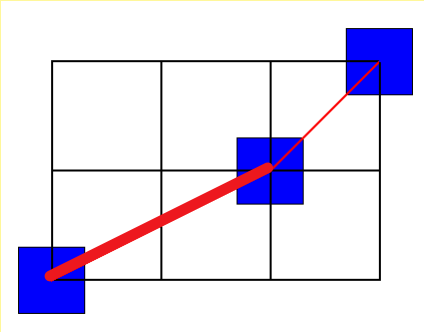
Newton polygon of $g(x)h(x)$ with respect to 2

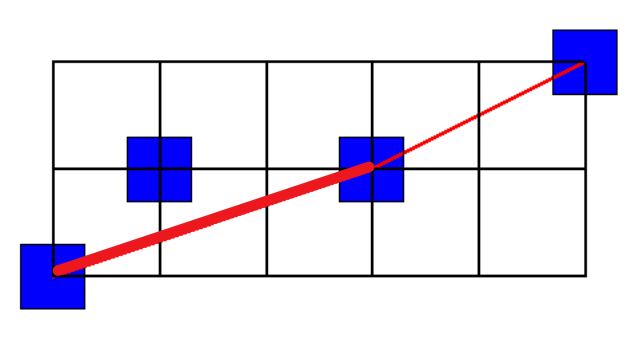


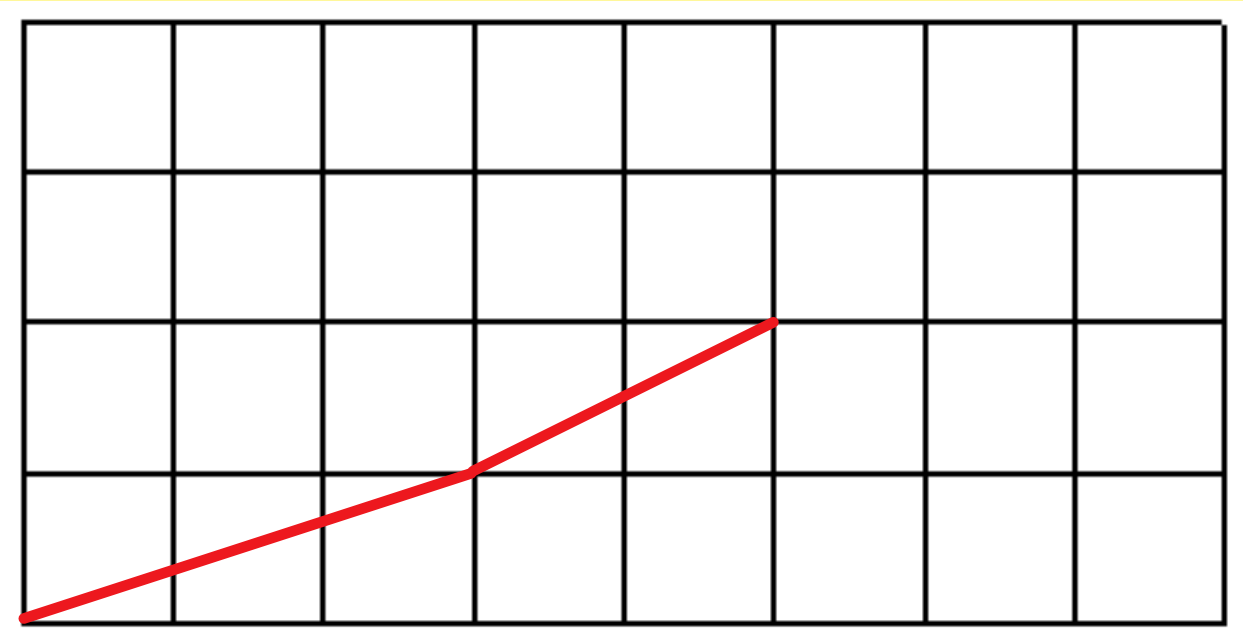




Newton polygon of $g(x)h(x)$ with respect to 2







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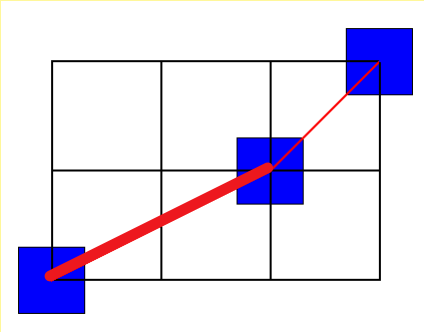
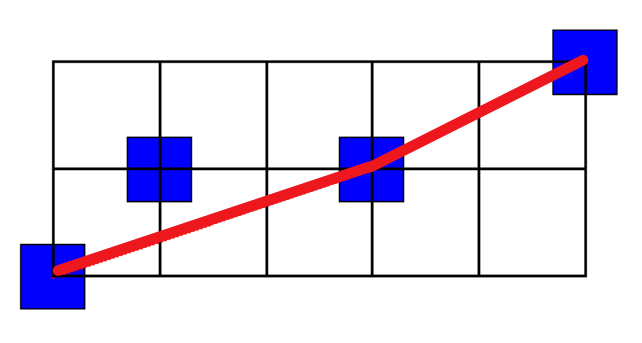
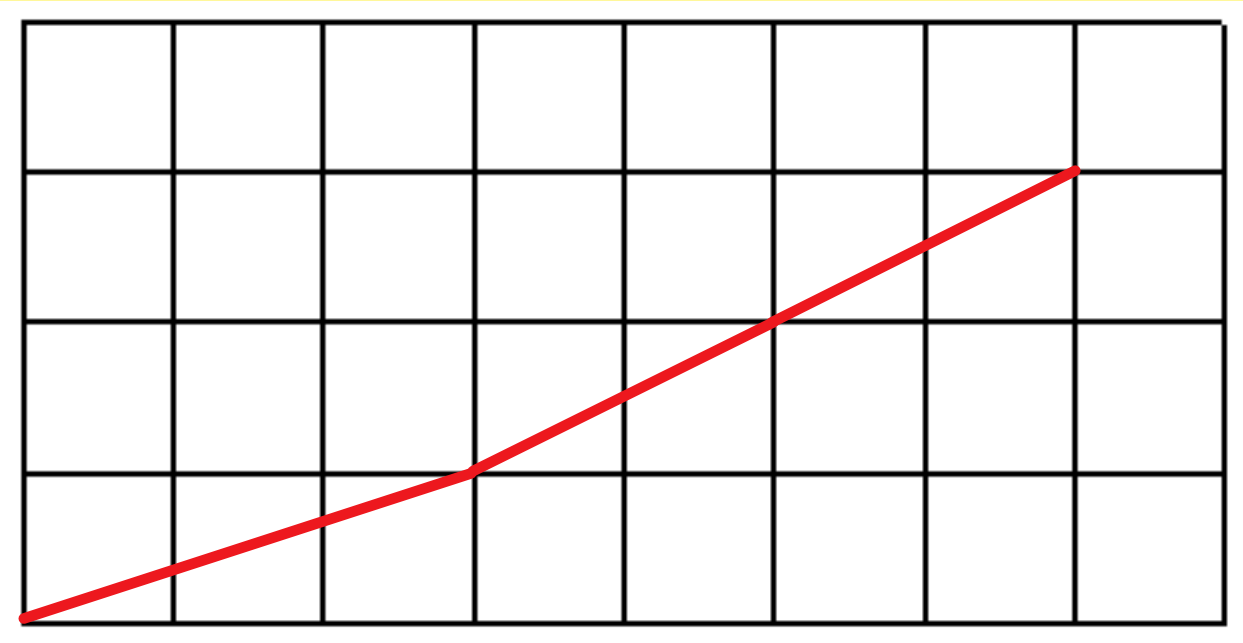
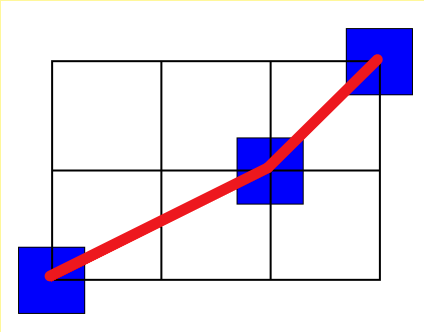
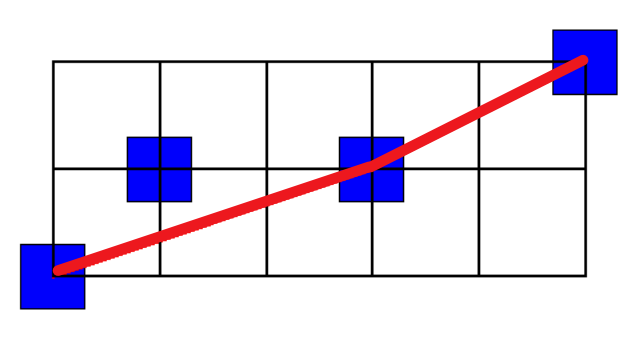


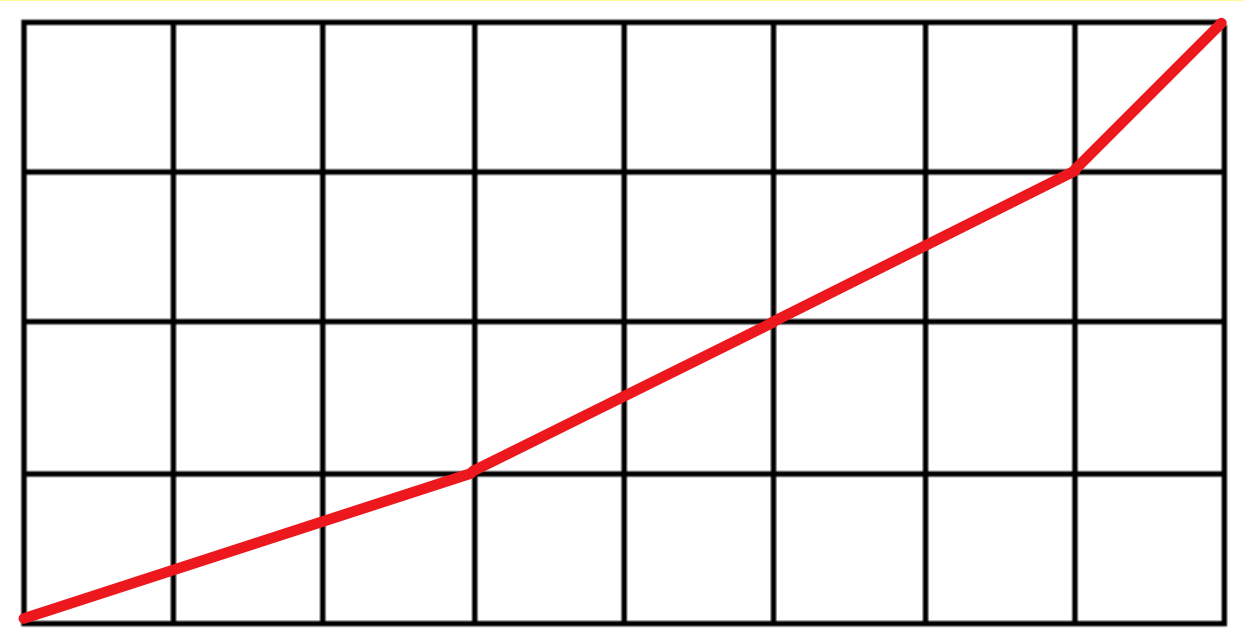




Newton polygon of $g(x)h(x)$ with respect to 2







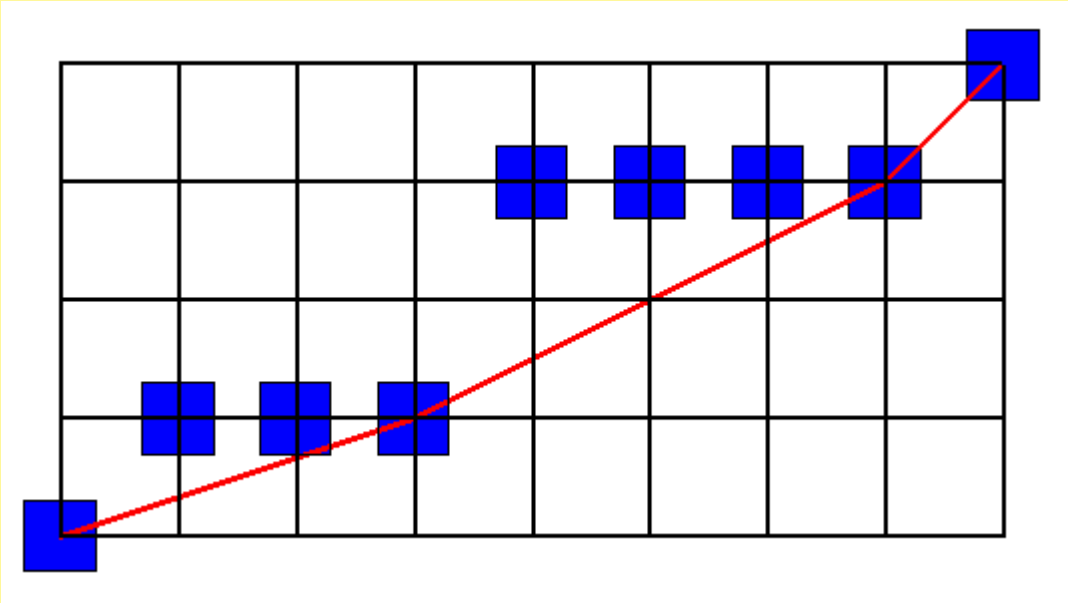
Newton polygon of $g(x)h(x)$ with respect to 2

$$g(x) = x^3 + 2x - 4, \;\; h(x) = x^5 - 6x^4 + 2x^2 - 12$$

$$g(x) = x^3 + 2x - 4, \;\; h(x) = x^5 - 6x^4 + 2x^2 - 12$$

$$f(x) = x^8 - 6x^7 + 2x^6 - 14x^5 + 24x^4 - 8x^3 - 8x^2 - 24x + 48$$

$$f(x) = x^8 - 6x^7 + 2x^6 - 14x^5 + 24x^4 - 8x^3 - 8x^2 - 24x + 48$$



Newton polygon of $f(x)$ with respect to 2

How is Dumas’ theorem used to establish irreducibility?

$$f(x) = x^{10} - 7x^8 + 14x^7 - 28x^5 - 49x^2 - 21$$

$$f(x) = x^{10} - 7x^8 + 14x^7 - 28x^5 - 49x^2 - 21$$

Why is $f(x)$ irreducible?

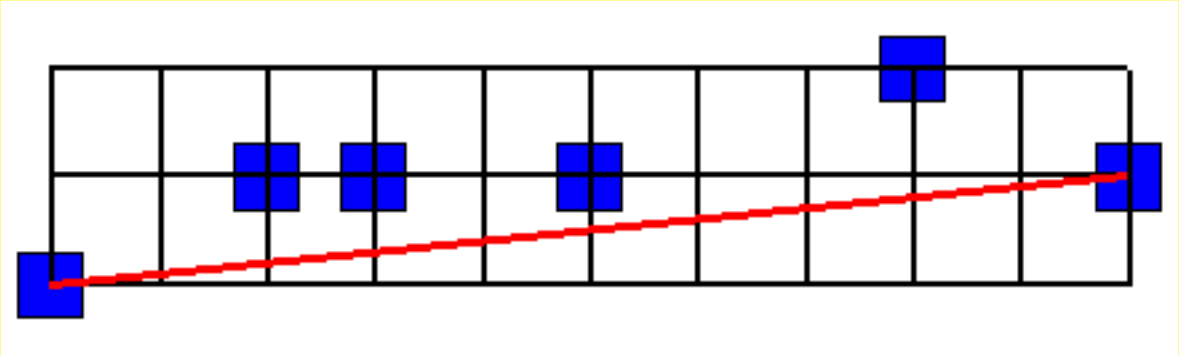
$$f(x) = x^{10} - 7x^8 + 14x^7 - 28x^5 - 49x^2 - 21$$

Why is $f(x)$ irreducible?

Eisenstein's Criterion applies

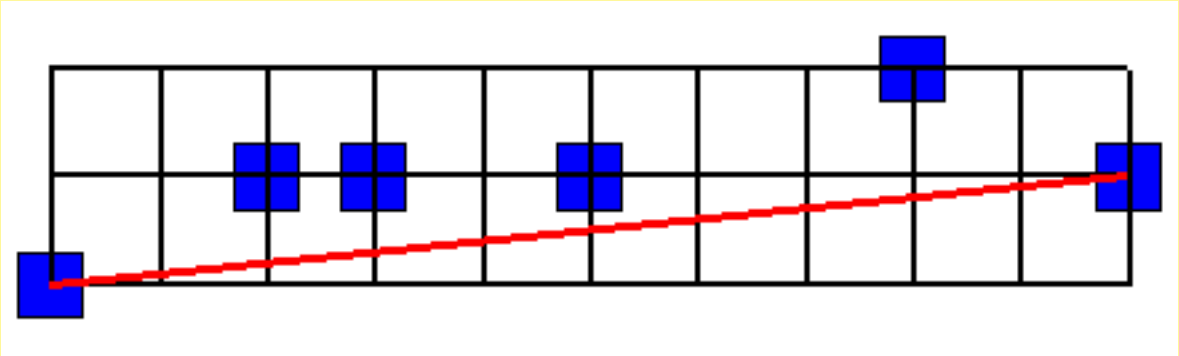
$$f(x) = x^{10} - 7x^8 + 14x^7 - 28x^5 - 49x^2 - 21$$

$$f(x) = x^{10} - 7x^8 + 14x^7 - 28x^5 - 49x^2 - 21$$



Newton polygon of $f(x)$ with respect to 7

$$f(x) = x^{10} - 7x^8 + 14x^7 - 28x^5 - 49x^2 - 21$$

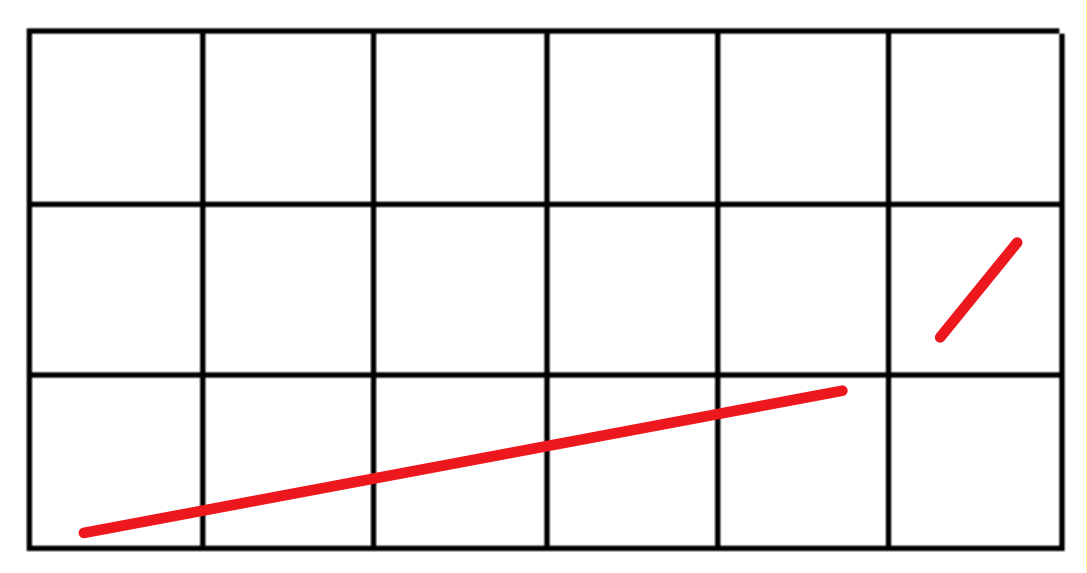


Newton polygon of $f(x)$ with respect to 7

What factors could $f(x)$ have?

$$f(x) = 2^{0}3^{0}x^{6} + 2^{3}3^{1}x^{5} + 2^{2}3^{1}x^{3} - 2^{1}3^{2}x + 2^{2}3^{2}$$

$$p = 2$$

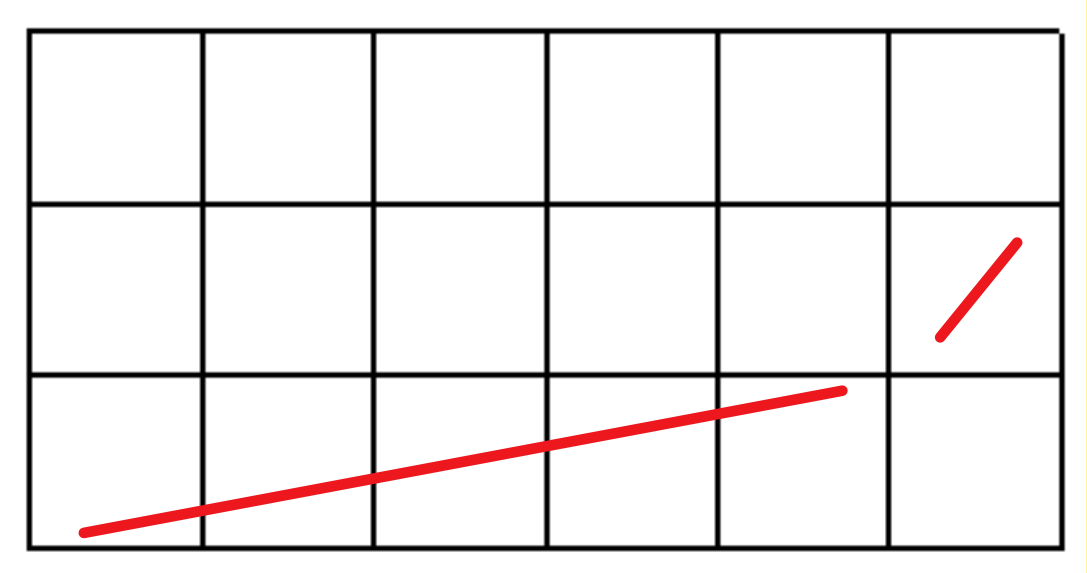


Newton polygon of $f(x)$ with respect to 2

$$f(x) = 2^{0}3^{0}x^{6} + 2^{3}3^{1}x^{5} + 2^{2}3^{1}x^{3} - 2^{1}3^{2}x + 2^{2}3^{2}$$

$$p = 2$$

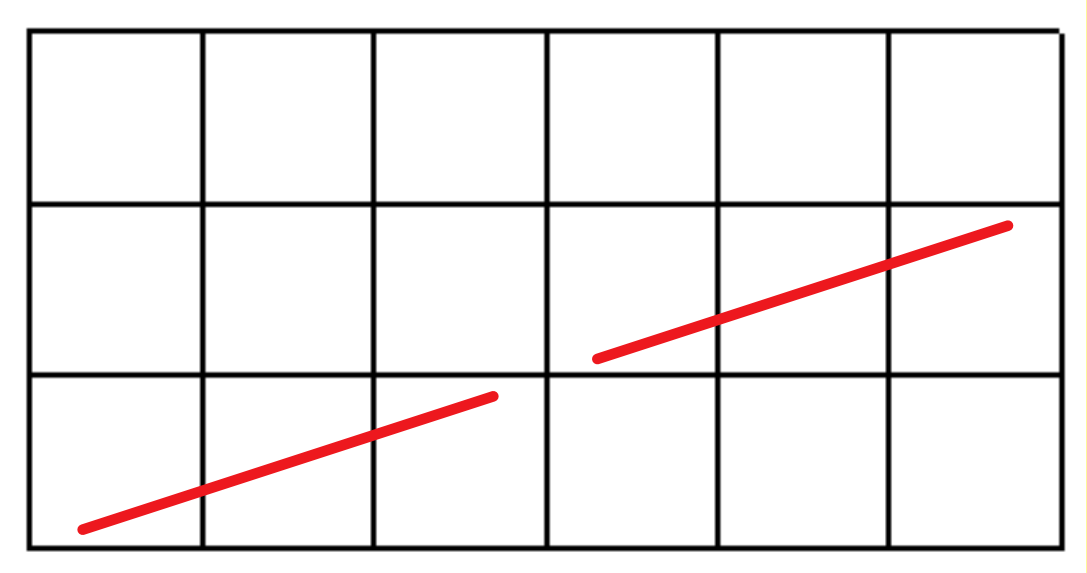
What factors could $f(x)$ have?



Newton polygon of $f(x)$ with respect to 2

$$f(x) = 2^0 3^0 x^6 + 2^3 3^1 x^5 + 2^2 3^1 x^3 - 2^1 3^2 x + 2^2 3^2$$

$$p = 3$$

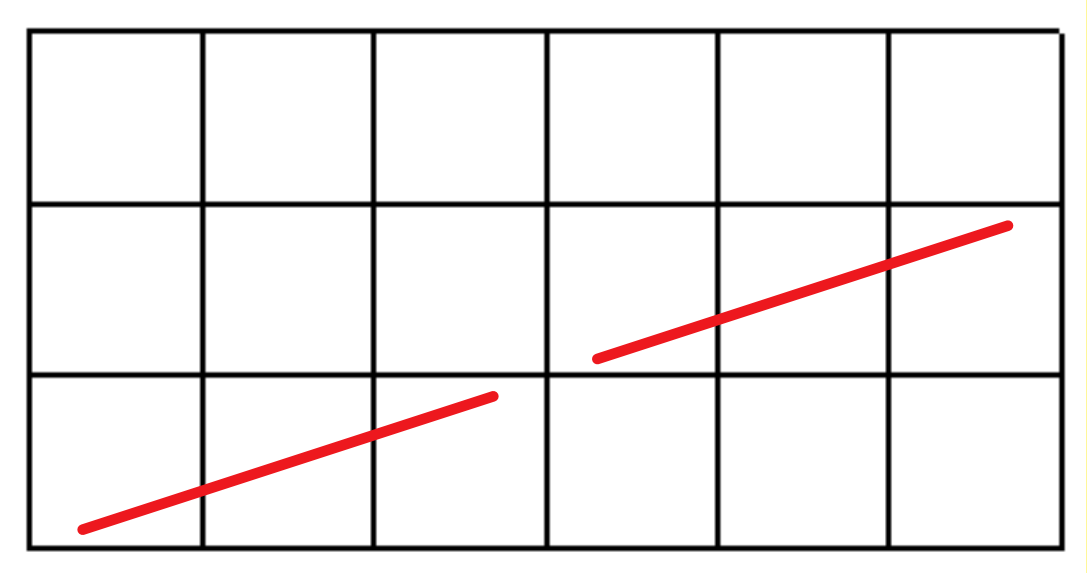


Newton polygon of $f(x)$ with respect to 3

$$f(x) = 2^0 3^0 x^6 + 2^3 3^1 x^5 + 2^2 3^1 x^3 - 2^1 3^2 x + 2^2 3^2$$

$$p = 3$$

What factors could $f(x)$ have?

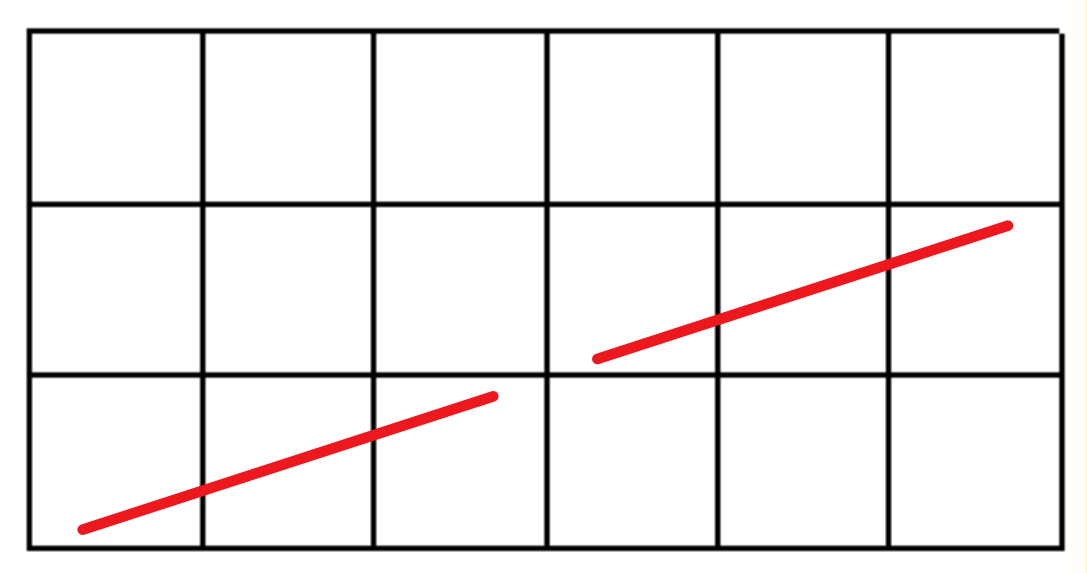


Newton polygon of $f(x)$ with respect to 3

$$f(x) = x^6 + 24\,x^5 + 12\,x^3 - 18\,x + 36$$

$f(x)$ is irreducible

What factors could $f(x)$ have?



Newton polygon of $f(x)$ with respect to 3