TOPICS IN FURTHER APPLICATIONS OF MATHEMATICAL LOGIC TO NUMBER THEORY

MATH 768 P
Fall 2005
Instructor: George F. McNulty

Resources: Hilbert's Tenth Problem
Yuri Matyasevich
MIT Press

Model Theory
C. C. Chang and H. Jerome Keisler
North-Holland

Recent journal articles of Bjorn Poonen
and others

Prerequisite: Graduate Standing

At the turn of the 20^{th} century, David Hilbert posed 23 problems scattered across the broad range of mathematics that, in large measure, touched the heart of mathematics as it has developed in the intervening years.

Hilbert's Tenth Problem concerned Diophantine equations. These are polynomial equations in several variables with integer coefficients.

Hilbert's Tenth Problem. *Given a Diophantine equation with any number of unknown quantities and with rational integral numerical coefficients: To devise a process according to which it can be determined by a finite number of operations whether the equation is solvable in rational integers.*

Hilbert's Tenth Problem was finally solved in 1969. A leading problem, still open, is whether a method can be found to determine whether given Diopanting equations have rational solutions. In the past few years substantial progress has been made on this problem, using methods from the theory of elliptic curves. This recent progress is the principal topic of this course. The students in this course will be brought to the leading edge of research in this field.

Of course, Hilbert was not the only one to offer conjectures that have challenged and motivated mathematicians. Emil Artin posed the following conjecture.

Artin's Conjecture. *For each positive integer d there exists a finite set Y of primes such that for every prime $p \notin Y$, every polynomial $f(t_0, \ldots, t_{n-1})$ of degree d over the field $\mathbb{Q}_p$ of p-adic numbers with zero constant term and $n > d^2$ has a nontrivial zero in $\mathbb{Q}_p$.*

This problem, like Hilbert's Tenth Problem. was solved using methods from mathematical logic. As time permits, this course will develop, also, those tools from mathematical logic that have proven useful in tackling Artin's Conjecture.

Grades will be determined on the basis of in-class and written presentations.