MATH 574: TEST 3

Name _____

Instructions and Point Values: Put your name in the space provided above. Make sure that your test has seven different pages including one blank page. Show all your work; the work should be sufficient for me to determine how you derived your answers. Calculators are NOT permitted on this test.

Point Values: Problems (1), (2), (3), (4), and (5) are each worth 16 points, Problem (6) is worth 18 points, and writing your name above correctly is worth 2 points.

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(1) Suppose $a_0 = 3$, $a_1 = 7$, and $a_n = 5a_{n-1} - 6a_{n-2}$ for $n \geq 2$. Find an explicit formula for a_n in terms of n.

Answer: $a_n =$

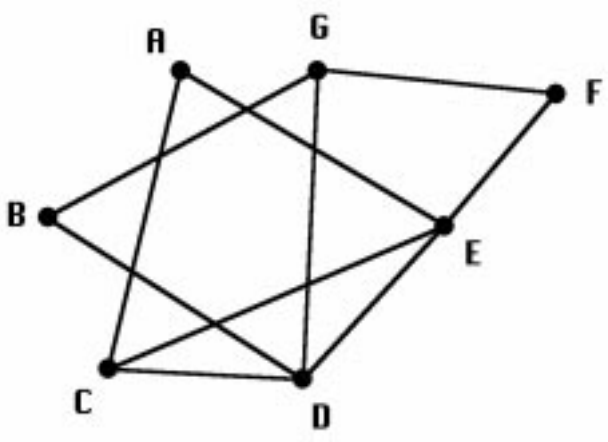
(2) Suppose $u_0 = 1$, $u_1 = 3$, $u_2 = 6$, and $u_n = 4u_{n-1} - 5u_{n-2} + 2u_{n-3}$ for $n \geq 3$. The characteristic polynomial for this recursion is

$$x^3 - 4x^2 + 5x - 2 = (x-1)^2(x-2).$$

Find an explicit formula for u_n in terms of n.

Answer: $u_n =$

(3) Given the graph G below, with 7 vertices and 10 edges as shown, answer the following questions.



(i) What is the degree of the vertex labelled C?

Answer: [ ]

(ii) Is the graph G complete? Explain your answer.

Answer: [ ]

(iii) What is the distance from vertex A to vertex G?

Answer: [ ]

(iv) There is an Euler path for this graph. What two vertices are the starting and ending vertices for such an Euler path?

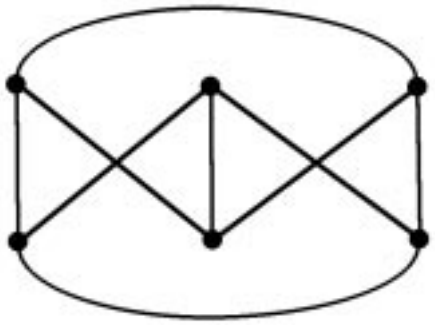
Answer: [ ] (answer with two letters in any order)

(4) Determine whether each of the graphs below is planar and circle the appropriate answers. Justify your answers in the space provided.

IMPORTANT: I am more interested in your justification than correct answers to the questions. Be sure your justifications are clear and complete.

HINT: One answer should be "Yes" and the other "No".

(a) Is the graph to the right planar? **YES** **NO** (circle one)



(b) Is the graph to the right planar? **YES** **NO** (circle one)

(5) The game of NIM is played with four stacks of coins having sizes 4, 7, 8, and 9. Thus, two players take turns, each turn consisting of removing any number of coins from any one stack. The last person to remove a coin wins.

(a) Is it best to move first or second in this game? Justify your answer with correct work.

Answer: [ ] (answer one of "first" or "second")

(b) Suppose the first player decides to remove 4 coins from the stack of size 9. Then the stacks have sizes 4, 7, 8, and 5. What is the best move for the second player to make in this situation? Again, you should justify your answer with correct work.

Answer: The second player should remove [ ] coins from the stack of size [ ].

(6) Recall that if $G = (V, E)$ is a graph, then we showed that

$$(*) \qquad \sum_{v \in V} \deg(v) = 2|E|.$$

In other words, the sum of all the degrees of the vertices is equal to twice the number of edges in a graph. Using $(*)$ and the fact that a tree on n vertices has $n - 1$ edges, explain why a tree on n vertices which has at least one vertex of degree 3 must have at least *three* vertices with degree 1.