Math 574: Test 3

Name ______________________________

Instructions and Point Values: Put your name in the space provided above. Make sure that your test has seven different pages including one blank page. Justify ALL answers with the proper work. Calculators are NOT permitted on this test.

Point Values: Each problem is worth 17 points (so 102 points are possible on this test).

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(1) Suppose $u_0 = 5$, $u_1 = 1$, and $u_n = 7u_{n-1} - 12u_{n-2}$ for $n \geq 2$. Find an explicit formula for u_n in terms of n.

Answer: $\boxed{u_n = \qquad\qquad}$

(2) Suppose $a_0 = 4$, $a_1 = 0$, $a_2 = 8$, and $a_n = a_{n-1} + a_{n-2} - a_{n-3}$ for $n \geq 3$. The characteristic polynomial for this recursion is

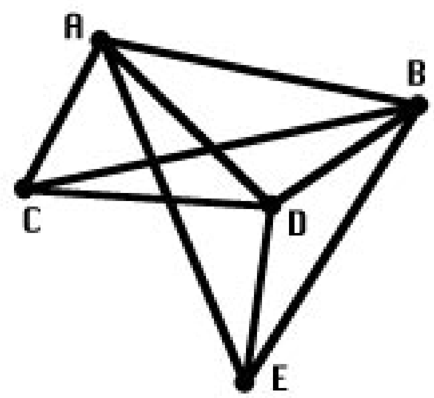
$$x^3 - x^2 - x + 1 = (x-1)^2(x+1).$$

(a) Find an explicit formula for a_n in terms of n.

Answer: $a_n =$

(b) What's the value of a_{1000}? Simplify your answer.

(3) Given the graph G below, with 5 vertices and 9 edges as shown, answer the following questions.



(i) What is the degree of the vertex labelled C?

Answer:

(ii) What is the distance from vertex C to vertex E?

Answer:

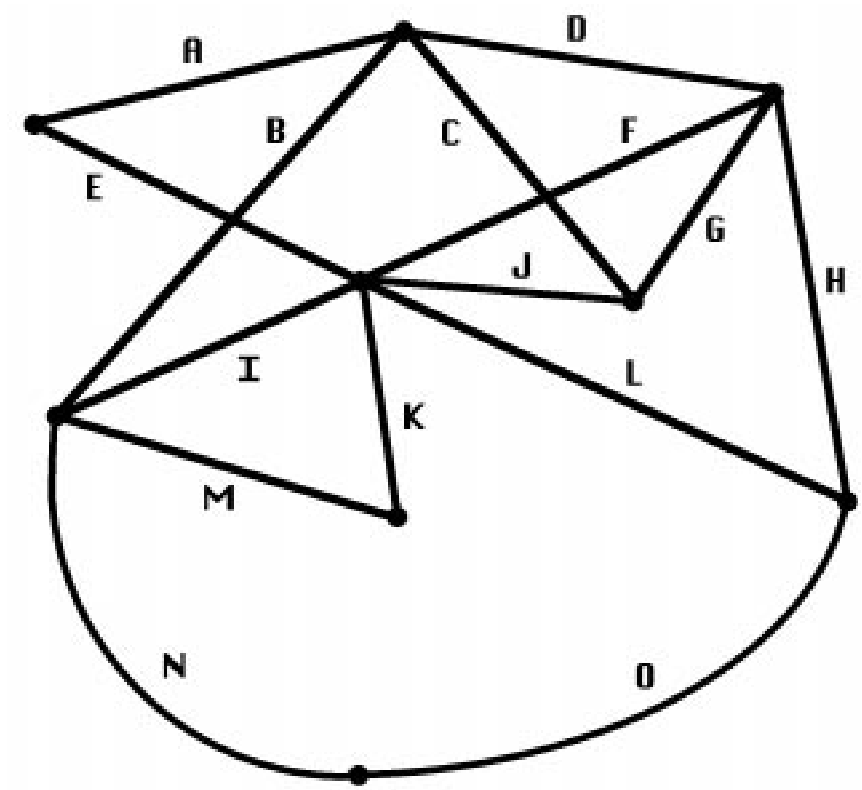
(iii) Is the graph G complete? Explain your answer.

Answer:

(iv) Is the graph G planar? Explain your answer.

Answer:

(4) The graph below has an Euler path. The edges are labelled by the 15 letters A, B, C, . . . , N, and O. Write a list of letters that produces an Euler path (for example, ABMKI . . . would indicate a path that goes through the edges A, B, M, K, I, . . . in that order).



Answer:

(5) The game of NIM is played with three stacks of coins having sizes 5, 9, and 12. Thus, two players take turns, each turn consisting of removing any number of coins from any one stack. The last person to remove a coin wins.

(a) Is it best to move first or second in this game? Justify your answer with correct work.

Answer: $\boxed{}$ (answer one of "first" or "second")

(b) Suppose the first player decides to remove 1 coin from the stack of size 12. Then the stacks have sizes 5, 9, and 11. What is the best move for the second player to make in this situation? Again, you should justify your answer with correct work.

Answer: The second player should remove $\boxed{}$ coins from the stack of size $\boxed{}$.

(6) Recall that if $G = (V, E)$ is a graph, then

$$(*) \qquad \sum_{v \in V} \deg(v) = 2|E|.$$

In other words, the sum of all the degrees of the vertices is equal to twice the number of edges in a graph. Recall also that a tree on n vertices has exactly $n - 1$ edges. Using $(*)$ show that if G is a tree on n vertices with no vertex of degree 2, then there are $> n/2$ vertices in G of degree 1. (Hint: Let k denote the number of vertices of degree 1 so that there are exactly $n - k$ vertices of degree at least 3.)