MATH 574: TEST 2

Name ____________________________

Instructions: Put your name in the space provided above. Make sure that your test has six different pages including one blank page. Show all your work; the work should be sufficient for me to determine how you derived your answers. Calculators are NOT permitted on this test.

Point Values: Problems (1), (2), (3), (4), and (5) are each worth 12 points, Problem (6) is worth 26 points, and Problem (7) is worth 14 points.

(1) Write all the 2-combinations of the set \(\{a, b, c, d\}\).
(2) A race involves 10 runners. First, second, and third place awards are made. How many possible outcomes are there for the awards? Simplify your answer.

Answer: 

(3) How many positive integers $\leq 1000$ are divisible by either 2 or 3 (possibly both)? Simplify your answer.

Answer: 

(4) A certain department at the University of South Carolina consists of exactly six women and six men. A committee of five people is to be formed having three of the women and two of the men on it. How many committees are possible? Simplify your answer.

Answer:  

(5) How many positive integers are there consisting of exactly ten digits all of which are distinct (i.e., each digit from \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} is used exactly once in each positive integer)? (Note: 0 is NOT to be used for the leading digit.)

Answer:  

(6) (a) State the Binomial Theorem.

(b) Calculate \( \sum_{k=0}^{n} \binom{n}{k} \) in closed form. Explain your answer.

(c) Calculate \( \sum_{k=0}^{n} \binom{n}{k}(-1)^k \) in closed form. Explain your answer.

(d) What is the value of

\[
\binom{100}{0} + \binom{100}{2} + \binom{100}{4} + \binom{100}{6} + \cdots + \binom{100}{100}
\]

In other words, what is \( \sum_{j=0}^{50} \binom{100}{2j} \)? Your answer should be expressed in the form \( a \times b^c \) where \( a, b, \) and \( c \) are explicitly stated numbers (one of them is 1). Justify your answer.

Answer: [ ]
(7) Prove that \( \sum_{k=0}^{n} \frac{\binom{n}{k}}{k + 1} = \frac{2^{n+1} - 1}{n + 1} \). This is a problem we did in class. You should show all your work here and not refer to what was done in class.