MATH 574, NOTES 7
PRACTICE PROBLEMS FOR TEST 2

(1) How many 4-permutations of the set \( \{1, 2, 3, 4, 5, 6\} \) contain the number 2?

(2) The number 12 written in base 2 is \((1100)_2\) which ends in 2 zeroes. In how many zeroes does the number 50! end when it is expressed in base 2?

(3) How many solutions are there to the equation

\[ x_1 + x_2 + x_3 = 10 \]

if each \( x_j \) is to be an integer from \( \{0, 1, 2, \ldots, 10\} \)?

(4) Let \( \mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_r \) be a complete list of distinct subsets of \( \{1, 2, \ldots, n\} \).

(a) Explain why \( r = 2^n \).

(b) If \( |\mathcal{A}_j| \) denotes the number of elements in the subset \( \mathcal{A}_j \), then what is the value of

\[ |\mathcal{A}_1| + |\mathcal{A}_2| + \cdots + |\mathcal{A}_r| \]?

(5) Calculate \( \sum_{k=1}^{n} \frac{(n)}{2^k} \) in closed form. (Note that the sum begins with \( k = 1 \) and not \( k = 0 \).)

(6) Prove that \( \sum_{k=0}^{n} \frac{(n)}{k+1} = \frac{2^{n+1} - 1}{n+1} \).

(7) (a) Recall that

\[ 1 + x + x^2 + \cdots = \frac{1}{1-x} \quad \text{if} \ |x| < 1. \]

Give a closed form expression for the sum \( 1 + 2x + 3x^2 + 4x^3 + \cdots \) that holds for \( |x| < 1 \).

(b) Calculate \( \sum_{k=0}^{\infty} \frac{k}{2^k} \).