MATH 574, NOTES 6
PERMUTATIONS AND COMBINATIONS

▶ **Definition:** A *k-permutation* of a set of distinct objects is an ordered arrangement of k objects from the set.

▶ **Definition:** A *k-combination* of elements from a set is an unordered selection of k elements from the set.

▶ **Notation:** $\binom{n}{k}$ denotes the number of k-combinations that exist for an n element set.

Examples:

(1) What are the 2-permutations of the set $\{1, 2, 3\}$?

(2) What are the 3-combinations of the set $\{1, 2, 3, 4\}$?

(3) What is number of k-permutations of an n element set?

(4) Show that $\binom{n}{k} = \dfrac{n!}{k!(n-k)!}$. Do this in two ways, by a counting argument and by a Calculus argument.

(5) Show that the coefficient of $x^k y^{n-k}$ in $(x+y)^n$ is $\binom{n}{k}$.

(6) Prove that $\binom{n}{k} = \binom{n}{n-k}$.

(7) Prove that $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$. Discuss Pascal's triangle.

(8) Show that $\displaystyle\sum_{k=0}^{n} \binom{n}{k} = 2^n$.

(9) Calculate $\displaystyle\sum_{k=0}^{n} (-1)^k \binom{n}{k}$ in closed form.

(10) Calculate $\displaystyle\sum_{k=0}^{n} k \binom{n}{k}$ in closed form.

(11) Calculate $\displaystyle\sum_{k=0}^{n} k^2 \binom{n}{k}$ in closed form.

(12) Calculate $\sum_{k=0}^{n} \binom{n}{k}^2$ in closed form.

(13) A race involves 8 runners. First, second, and third place awards are made. How many possible outcomes are there for the awards?

(14) A committee of 5 people is to be formed from a group of 10 people. How many committees are possible?

(15) How many subsets of $\{a, b, c, d, e\}$ contain 3 elements.

(16) A path is taken from the origin $(0, 0)$ in the plane to the point $(8, 12)$. Each step in the path consists of moving one unit to the right or one unit up. How many such paths are possible?