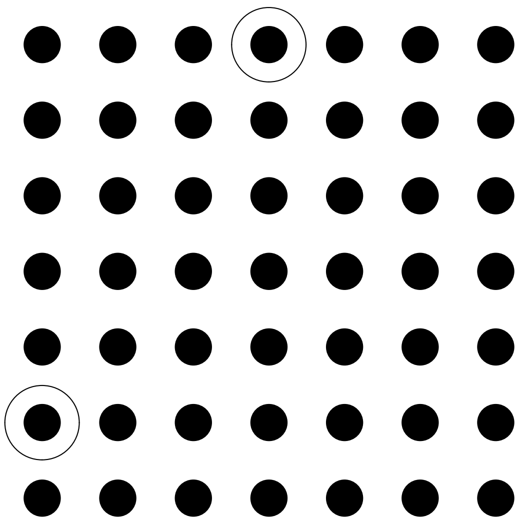
MATH 532, 736I: MODERN GEOMETRY

Name ______________________________

Practice Test #1

(1) State the axioms for a finite affine plane of order n. (Number or name the axioms so you can refer to them.)

(2) Two points have been circled in the 7×7 array of points below. Using the model for a finite affine plane of order 7 discussed in class, finish circling the points that belong to the same line as the given circled points.



(3) Still consider the array above as a model of an affine plane of order 7 as discussed in class. Find the equation of the line passing through the two circled points above. Put your answer in the form $y \equiv mx + k \pmod 7$ where m and k are among the numbers $0, 1, 2, 3, 4, 5,$ and 6.

Answer:

(4) Using only the results about affine planes below and the axioms you stated in Problem 1 (and referring to them whenever you use them), fill in the boxes with the appropriate numbers and finish proving that in an affine plane of order n, each point has exactly $n+1$ lines passing through it. This is a proof you were to have memorized from class. Note that each result below involves a point and a line. Whenever you use one of the results below, be sure to clarify what point and line you are using with the result.

Result 1: *If ℓ is a line with exactly n points on it and A is a point not on ℓ, then there are exactly $n+1$ lines passing through A.*

Result 2: *If A is a point with exactly $n+1$ lines passing through it and ℓ is a line with A not on ℓ, then there are exactly n points on ℓ.*

Proof. Let P be a point. By $\boxed{\qquad\qquad}$, there is a line ℓ with exactly n points on it. If P is not on ℓ, then we can use $\boxed{\qquad\qquad}$ (with point $\boxed{\quad}$ and line $\boxed{\quad}$) to deduce that there are exactly $n+1$ lines passing through P. Suppose then that P is on ℓ. From $\boxed{\qquad\qquad}$, we know that there are 4 distinct points, no 3 of which are collinear. Since no 3 of the 4 points are collinear, at most 2 of them can be on ℓ. Thus, at least 2 of them are not on ℓ. Let A and B denote 2 of these 4 points with neither A nor B on ℓ. Let C and D denote the other 2 of these 4 points. By $\boxed{\qquad\qquad}$, there is a line ℓ_1 through A and C and a line ℓ_2 through A and D. Since A, C, and D are not collinear, the lines ℓ_1 and ℓ_2 are distinct. Note that $P \neq A$ since P is on ℓ and A is not. By $\boxed{\qquad\qquad}$, the point P cannot be on both ℓ_1 and ℓ_2. Let ℓ' be one of ℓ_1 and ℓ_2 with P not on ℓ'. Since no 3 of A, B, C, and D are collinear, B is not on the line ℓ'.

(Complete the proof. Whenever you use one of the results above, clarify what point and line you are using with the result.)

Part II. Problem 1 is worth 9 points, Problem 2 is worth 15 points, Problems 3 and 4 are each worth 8 points, and Problem 5 is worth 14 points. The problems in this section all deal with an axiomatic system consisting of the following axioms.

Axiom 1. There exist at least one point and at least one line with the point not on the line.

Axiom 2. Given any 2 distinct lines, there exists exactly one point on both lines.

Axiom 3. Given any point, there exist at least two lines passing through the point.

(1) Justify that the axiomatic system is consistent.

(2) Justify that the axiomatic system is independent.

(3) Justify that the axiomatic system is *not* complete.

(4) What is the dual of Axiom 3? (Just write it down.)

(5) Prove that in this axiomatic system there exist at least 3 distinct points.