MATH 532, 736I: MODERN GEOMETRY

Name ______________________________

Test #2 (1993)

Show All Work
Points: Part I (12 pts each), Part II (16 pts each)

Part I. Each of the following is something or part of something you were asked to memorize.

(1) Theorems are listed on the last page of this test. They may or may not have the numbering that you are accustomed to them having from class. Prove Theorem 3 using Theorem 1 (but not Theorem 2 or Theorem 4).

(2) Let A, B, and C be 3 noncollinear points. Let M_A be the midpoint of $\overline{BC}$ and let M_B be the midpoint of $\overline{AC}$. Let D be the intersection of the (extended) altitudes of ΔABC. Let $N = (A + B + C + D)/4$. Prove that the distance from N to M_A is the same as the distance for N to M_B. This is part of the $9-$point circle theorem, so you should not make use of the $9-$point circle theorem in doing this problem.

(3) On the second to the last page is a drawing consisting of 2 triangles ΔABC and $\Delta A'B'C'$ which are perspective from the line passing through X, Y, and Z. Suppose we want to prove that these triangles are also perspective from a point. Let P be the point of intersection of the lines $\overleftrightarrow{AA'}$ and $\overleftrightarrow{BB'}$. To show that $\overleftrightarrow{CC'}$ also passes through P, we can consider two triangles which we know to be perspective from a point. The vertices of these two triangles (not necessarily the edges) and the point they are perspective from are all in the drawing. Determine the two triangles and the point they are perspective from and indicate your answers in the boxes below. (No work necessary.)

Triangles $\boxed{}$ and $\boxed{}$ are perspective from point $\boxed{}$.

(Note: You can complete the boxes to make a sentence which is true, but that will not necessarily insure that you have a correct answer. A correct answer must correspond to establishing that $\overleftrightarrow{CC'}$ passes through P as discussed above.)

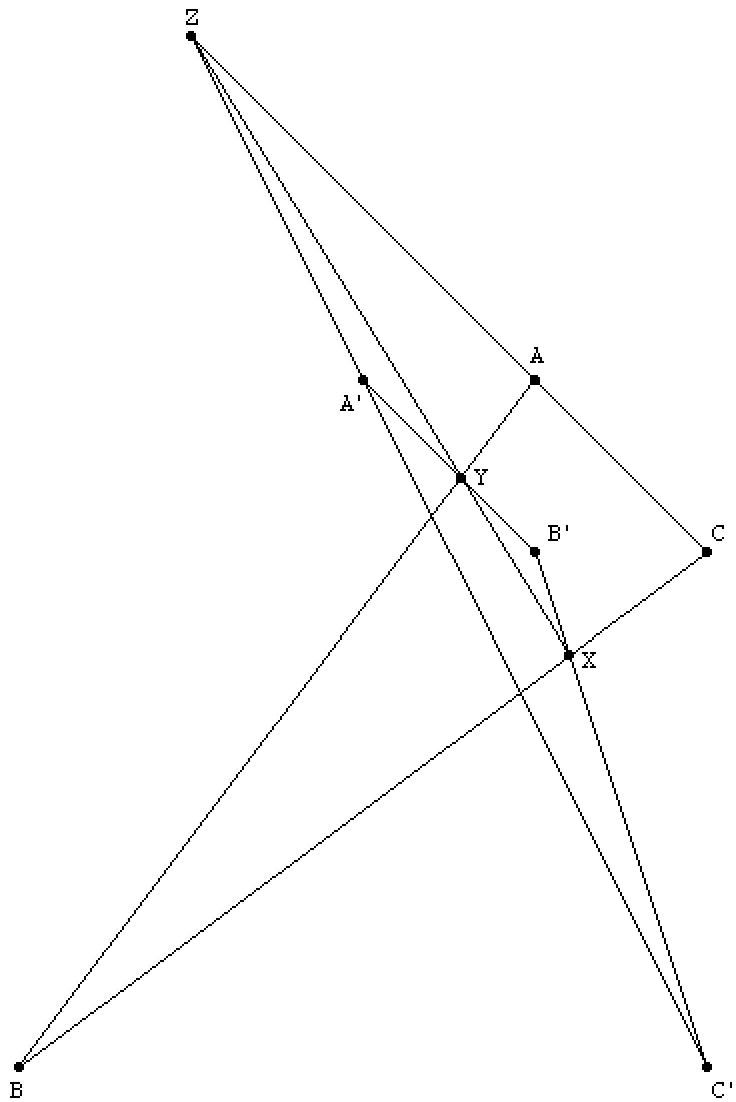
Part II. Each problem in this section is either related to the homework, related to something you were to have memorized, or just plain new. Since each problem is worth 16 points, be sure to make your answers clear and precise.

(1) Given ΔABC, let D be the midpoint of $\overline{BC}$, let E be the midpoint of $\overline{AC}$, let F be the midpoint of $\overline{AB}$, and let G be the midpoint of $\overline{DE}$. Using vectors, prove that C, F, and G are collinear and that G is the midpoint of $\overline{CF}$.

(2) Let the function $f(x, y)$ be defined as follows. First f rotates each point (x, y) about the point $(0, 0)$ by $\pi/2$, then it takes the result and rotates it about $(1, 0)$ by π, and then it takes that result and rotates it about the point $(1, 1)$ by $\pi/2$. Determine whether f is a translation or a rotation and write it in the form T_B or $R_{\alpha,A}$ giving specific values for B, α, and A if appropriate (i.e., if $f = T_B$, tell me what B is, and if $f = R_{\alpha,A}$, tell me what α and A are).

(3) Let P_1, P_2, P_3, and P_4 be 4 (not necessarily distinct) points. Let A and B be arbitrary points. Beginning with $A_0 = A$, for $j \in \{1, 2, 3, 4\}$, define A_j as the point you get by rotating A_{j-1} about P_j by π. Set $Q_1 = P_3$, $Q_2 = P_4$, $Q_3 = P_1$, and $Q_4 = P_2$. Beginning with $B_0 = B$, for $j \in \{1, 2, 3, 4\}$, define B_j as the point you get by rotating B_{j-1} about Q_j by π. Prove that A, B, A_4, and B_4 either all lie on a line or they are the vertices of a parallelogram.

(4) Suppose that α and β are angles with $\alpha + \beta = \pi$. Let A and B be distinct points. By a theorem from class, we know that there is a point C such that $R_{\beta,B}R_{\alpha,A} = R_{\pi,C}$. Explain why C must be on the circle centered at $\frac{1}{2}(A+B)$ passing through A and B. (Hint: You may want to use a geometrical interpretation of $R_{\beta,B}R_{\alpha,A}$ rather than matrices.)



Part I, Problem 3

INFORMATION PAGE

Theorem 1: *Let A and B be distinct points. Then C is a point on line $\overleftrightarrow{AB}$ if and only if there is a real number t such that*

$$C = (1-t)A + tB.$$

Theorem 2: *If A, B, and C are points and there are real numbers x, y, and z not all 0 such that*

$$x + y + z = 0 \quad \textit{and} \quad xA + yB + zC = 0,$$

then A, B, and C are collinear.

Theorem 3: *If A, B, and C are collinear, then there are real numbers x, y, and z not all 0 such that*

$$x + y + z = 0 \quad \textit{and} \quad xA + yB + zC = 0.$$

Theorem 4: *If A, B, and C are not collinear and if there are real numbers x, y, and z such that*

$$x + y + z = 0 \quad \textit{and} \quad xA + yB + zC = 0,$$

then $x = y = z = 0$.

$$T_{(a,b)} = \begin{pmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix}$$

$$R_{\theta,(x_1,y_1)} = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & x_1(1-\cos(\theta)) + y_1\sin(\theta) \\ \sin(\theta) & \cos(\theta) & -x_1\sin(\theta) + y_1(1-\cos(\theta)) \\ 0 & 0 & 1 \end{pmatrix}$$

$$T_{(a,b)} = R_{\pi,(a/2,b/2)} R_{\pi,(0,0)}$$