MATH 532, 736I: MODERN GEOMETRY

Test 1, Spring 2019 Name ______________________________

Show All Work

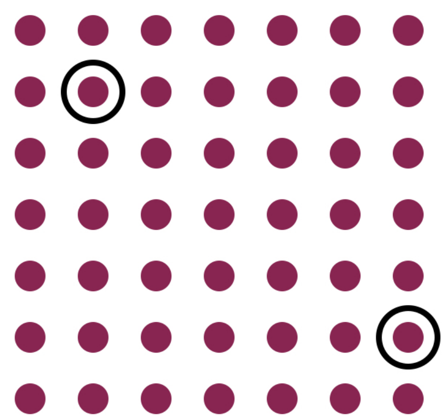
Instructions: This test consists of 3 pages of problems. Put your name at the top of this page and at the top of the first page of the packet of blank paper given to you. Work each problem in the packet of paper unless it is indicated that you can or are to do the work below. Show ALL of your work. Do NOT use a calculator.

Points: Part I (50 pts), Part II (50 pts)

Part I. The point value for each problem appears to the left of each problem. In Problem 5, I will assume you are using the axioms as you state them in your answer to Problem 1 below.

10 pts (1) Draw a model for a finite PROJECTIVE plane of order 3. Be sure to clearly draw every point and every line in your model. If I look at your model, I should be able to tell where each of your points and lines are. In particular, make sure that each line you draw cannot be mistaken for two lines. (Note: you do not need to label your points and lines.)

10 pts (2) Two points have been circled in the 7×7 array of points to the right. Using the model for a finite affine plane of order 7 discussed in class, finish circling the points that belong to the same line as the given circled points. (You do not need to use the packet of blank paper for this problem.)



12 pts (3) Consider the points $(4, 14)$ and $(20, 3)$ in a 31×31 array of points for our model of a finite affine plane of order 31. Find the equation of the line passing through these two points. Put your answer below in the form $y \equiv mx + k \pmod{31}$ where m and k are among the numbers $0, 1, 2, \ldots, 30$. Be sure to show your work in the packet provided with this test. You will not get credit for a correct answer without correct work.

Answer: $\boxed{}$ (Make sure m and k are in $\{0, 1, 2, \ldots, 30\}$.)

18 pts (4) Recall that the axioms for a finite projective plane are

Axiom P1: There exist at least 4 points no 3 of which are collinear.
Axiom P2: There exists at least 1 line with exactly $n+1$ (distinct) points on it.
Axiom P3: Given 2 distinct points, there is exactly 1 line that they both lie on.
Axiom P4: Given 2 distinct lines, there is at least 1 point on both of them.

Fill in the boxes below to complete a proof of the result stated below. Note that this proof has some different wording than from previous tests, and you need to make the argument given here correct and so cannot rely completely on what was in blanks on the prior tests for the course.

> Result: *If ℓ is a line with exactly $n+1$ points on it in a finite projective plane of order n and A is a point not on ℓ, then there exist at least $n+1$ distinct lines passing through A.*

This is part of a result done in class and that you were to have learned for this test. Use only the axioms stated above to complete the proof.

Proof. Let $P_1, P_2, \ldots, P_{n+1}$ be the $n+1$ distinct points on ℓ. Since

$\boxed{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$

we have that $A \neq P_j$ for each $j \in \{1, 2, \ldots, n+1\}$. By $\boxed{\qquad\qquad\qquad}$, there is a line ℓ_j passing through A and P_j for each $j \in \{1, 2, \ldots, n+1\}$. Since $\boxed{\qquad\qquad\qquad\qquad}$ and A is on ℓ_j, we see that $\ell_j \neq \ell$ for each $j \in \{1, 2, \ldots, n+1\}$. We justify next that

$\boxed{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$.

Assume $\boxed{\qquad\qquad}$ for some i and j in $\{1, 2, \ldots, n+1\}$ with $i \neq j$. Then the two points $\boxed{\qquad\qquad\qquad}$ are both on ℓ_i. Since these two points are distinct, $\boxed{\qquad\qquad\qquad}$ implies that there is $\boxed{\qquad\qquad\qquad\qquad}$ passing through them. Since $\boxed{\qquad\qquad\qquad}$ are two points that are both on ℓ_i and are both on ℓ, we deduce $\boxed{\qquad\qquad}$. This contradicts that $\boxed{\qquad\qquad\qquad\qquad}$. Thus, our assumption is wrong and the lines $\ell_1, \ell_2, \ldots, \ell_{n+1}$ are different. This finishes the proof that there are at least $n+1$ distinct lines passing through A. ■

Part II. The point values appear to the left of each problem. The problems in this section all deal with an axiomatic system consisting of the following axioms.

Axiom 1. There exists at least one line.

Axiom 2. Given any line, there is another line which is parallel to it.

Axiom 3. Given any line, there is another line which intersects it.

8 pts (1) Justify that the axiomatic system is consistent. Use the blank paper.

6 pts (2) Justify that the axiomatic system is *not* complete. Include some brief explanation, in complete English sentences, for your answer. Use the blank paper.

12 pts (3) Justify that the axiomatic system is independent. Use the blank paper.

8 pts (4) Write the dual of Axiom 2 below. (Write in clear English. Do not refer to parallel points.)

Dual of Axiom 2.

16 pts (5) Prove that there are at least 4 lines in any model for this axiomatic system. Use the blank paper.