MATH 532, 736I: MODERN GEOMETRY

Name ______________________________

Test #1

Show All Work
Points: Part I (60 pts), Part II (40 pts)

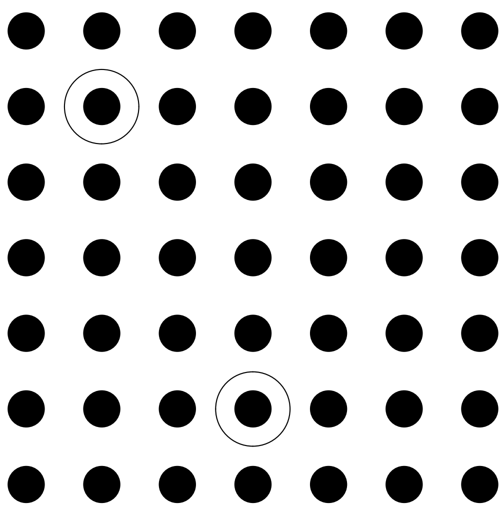
Part I. Each problem in this section is worth 10 points. The last problem, Problem 6, is one of the proofs that you were to have memorized. In Problem 6, I will assume you are using the axioms as you state them in your answer to Problem 2 below.

(1) State the axioms for a finite projective plane of order n.

(2) State the axioms for a finite affine plane of order n. (Number or name the axioms so you can refer to them.)

(3) Give a model for a finite projective plane of order 2. Be sure to clearly mark every point and clearly draw every line in your model.

(4) Two points have been circled in the 7×7 array of points below. Using the model for a finite affine plane of order 7 discussed in class, finish circling the points that belong to the same line as the given circled points.



(5) Consider the points $(3, 8)$ and $(9, 10)$ in a 17×17 array of points for our model of a finite affine plane of order 17. Find the equation of the line passing through these two points. Put your answer in the form $y \equiv mx + b \pmod{17}$ where m and b are among the numbers $0, 1, 2, \ldots, 16$.

(6) Using only the result below and the axioms you stated in Problem 2 (and referring to them whenever you use them), prove that in an affine plane of order n, for each line ℓ, there are at least $n-1$ lines which do not intersect ℓ. This is part of a proof you were to have memorized for class. (More precisely, the problem you were to have memorized for class involved showing that there are "exactly" $n-1$ such lines. I have not used the word "exactly" in my statement of the problem above.)

Theorem: *In an affine plane of order n, each line contains exactly n points.*

Part II. The problems in this section all deal with an axiomatic system consisting of the axioms below. Be sure to answer the questions being asked. For example, if you are giving a model to justify your answer in Problem 1 below, make sure you also state whether your answer is, “Yes” or “No.” Problems 1 and 3 are worth 6 points each, Problems 2 and 5 are worth 10 points each, and Problem 4 is worth 8 points.

Axiom 1. There exist at least 4 distinct points.

Axiom 2. Every line has at least 3 distinct points on it.

Axiom 3. Given any 2 distinct points, there exists exactly one line passing through the 2 points.

(1) Is the axiomatic system consistent? Justify your answer.

(2) Is the axiomatic system independent? Justify your answer.

(3) Is the axiomatic system complete? Justify your answer.

(4) We say that a model of an axiomatic system satisfies the principle of duality if for any theorem which is true in the model, the dual of the theorem also holds in the model.

(a) Explain why there are models which satisfy the axioms of the axiomatic system and which satisfy the principle of duality.

(b) Explain why there are models which satisfy the axioms of the axiomatic system but which do not satisfy the principle of duality.

(5) Suppose we add to the axiomatic system a fourth axiom:

Axiom 4. There exist 3 noncollinear points.

Prove that there exist 4 points no 3 of which are collinear.