Math 532: Quiz 4

Name ______________________________

Using only the axioms and lemmas on the reverse side of this paper, fill in the boxes to finish the proof that in an affine plane of order n, each point has exactly $n+1$ lines passing through it. Note that the lemmas and their numbering are not necessarily what you are accustomed to.

Proof: Let A be an arbitrary point. By $\boxed{}$, there is a line ℓ with exactly n points on it. If A is not on ℓ, then explain why A has exactly $n+1$ lines passing through it. Be clear (clarify whatever points and lines you are using).

$\boxed{}$

Now, consider the case that A is on ℓ. By $\boxed{}$, there are at least two points B and C not on ℓ. By $\boxed{}$, there are exactly $\boxed{}$ lines passing through B and exactly $\boxed{}$ lines passing through C. In particular, by Lemma 1, there are at least 3 lines passing through C. By Axiom A3, there is exactly one line passing through $\boxed{}$ and exactly one line passing through $\boxed{}$. Therefore, there is at least one line, say ℓ', passing through C that does not pass through $\boxed{}$. Explain why ℓ' has exactly n points on it. Be clear (as noted above).

$\boxed{}$

Finish the proof. Again, be clear (as noted above).

$\boxed{}$

Axioms for an Affine Plane

(you will need to know these for a test)

Axiom A1. There exist at least 4 distinct points no 3 of which are collinear.

Axiom A2. There exists at least 1 line with exactly n points on it.

Axiom A3. Given any 2 distinct points, there exists exactly one line passing through the 2 points.

Axiom A4. Given any line ℓ and any point P not on ℓ, there is exactly 1 line through P that does not intersect ℓ.

Two Lemmas for Affine Planes

(these would be given to you for a test on the proof given on the previous page)

Lemma 1. *An affine planes has order* ≥ 2.

Lemma 2. *If ℓ is a line with exactly n points on it and A is a point not on ℓ, then there are exactly $n+1$ lines passing through A.*

Lemma 3. *If A is a point with exactly $n+1$ lines passing through it and ℓ is a line with A not on ℓ, then there are exactly n points on ℓ.*