Math 532: Quiz 2

Name _____

Axiom P1: There exist at least 4 points no 3 of which are collinear.

Axiom P2: There exists at least 1 line with exactly $n+1$ (distinct) points on it.

Axiom P3: Given 2 distinct points, there is exactly 1 line that they both lie on.

Axiom P4: Given 2 distinct lines, there is at least 1 point on both of them.

In this problem, you are to help provide a proof for the following result done in class:

> Result: *If ℓ is a line with exactly $n+1$ points on it in a finite projective plane of order n and A is a point not on ℓ, then there exist exactly $n+1$ lines passing through A.*

You should not use any results that we have already established about finite projective planes. Where I have left room for explanations, be sure to justify your comments using the axioms given above. Note that the quiz includes needed work to be done on the back of this page.

Proof: Let $P_1, P_2, \ldots, P_{n+1}$ be the $n+1$ points on ℓ. In the box below, indicate which axiom is used to get lines $\ell_1, \ell_2, \ldots, \ell_{n+1}$ passing through A and explain what these lines are (i.e., how you are using the axiom).

<div style="border:1px solid">

Box 1
</div>

In the next box, explain why each of the $n+1$ lines $\ell_1, \ell_2, \ldots, \ell_{n+1}$ is not equal to ℓ.

<div style="border:1px solid">

Box 2
</div>

In the next box, justify that the $n+1$ lines $\ell_1, \ell_2, \ldots, \ell_{n+1}$ are different. In other words, justify that if i and j are in $\{1, 2, \ldots, n+1\}$ with $i \neq j$, then $\ell_i \neq \ell_j$. You should use one of the axioms and the information from Box 2 above. Clarify where you use them.

<div style="border:1px solid">

Box 3
</div>

(turn over to complete the quiz)

The above justifies that there are at least $n + 1$ different lines passing through A. To finish the proof, we need to show that there are no more lines passing through A. Let ℓ' be a line passing through A. Justify that $\ell' = \ell_j$ for some j in $\{1, 2, \ldots, n + 1\}$. Your justification should refer to more than one of the axioms.

Box 4

The above shows that there are at least $n + 1$ lines passing through A and that there are no more than $n + 1$ lines passing through A. Hence, there are exactly $n + 1$ lines passing through A. This completes the proof of the result. ∎