1. **Homework:** Problem Sheet on Vector Notation  
   Quiz: 03/27/02, Wednesday

2. **Theorem 1.** Let \( A \) and \( B \) be distinct points. Then \( C \) is on \( \overrightarrow{AB} \) if and only if there is a real number \( t \) such that \( C = (1 - t)A + tB \).

   **Basic Ideas of Proof:**
   - \( \overrightarrow{AC} = t \overrightarrow{AB} \)
   - \( C - A = t(B - A) \)

3. **Comment:** In Theorem 1,
   \[
   \frac{t}{1 - t} = \pm \frac{\text{length of } \overrightarrow{AC}}{\text{length of } \overrightarrow{CB}}
   \]
   where a plus sign occurs on the right if and only if \( C \) is between \( A \) and \( B \) and one denominator is 0 if and only if the other denominator is 0.

   **Basic Idea of Proof:** Consider three cases depending on the position of \( C \) relative to \( A \) and \( B \).

4. **Theorem 2.** If \( A, B \) and \( C \) are collinear, then there exist real numbers \( x, y, \) and \( z \) not all 0 such that
   \[ x + y + z = 0 \quad \text{and} \quad xA + yB + zC = \overrightarrow{0}. \]

   **Basic Ideas of Proof:**
   - If \( A = B \), take \( x = 1, y = -1, \) and \( z = 0. \)
   - Otherwise, use Theorem 1 and take \( x = 1 - t, y = t, \) and \( z = -1. \)

5. **Theorem 3.** If \( A, B \) and \( C \) are points and there exist real numbers \( x, y, \) and \( z \) not all 0 such that
   \[ x + y + z = 0 \quad \text{and} \quad xA + yB + zC = \overrightarrow{0}, \]
   then \( A, B \) and \( C \) are collinear.

   **Basic Ideas of Proof:**
   - Relabel so \( x \neq 0. \)
   - Deduce \( A = (-y/x)B + (-z/x)C \) and \( 1 = -y/x - z/x. \)
   - Take \( t = -z/x \) so that \( 1 - t = -y/x. \)
   - Use Theorem 1.

6. **Theorem 4.** If \( A, B \) and \( C \) are not collinear and there exist real numbers \( x, y, \) and \( z \) such that
   \[ x + y + z = 0 \quad \text{and} \quad xA + yB + zC = \overrightarrow{0}, \]
   then \( x = y = z = 0. \)

   **Basic Idea of Proof:** This is a rewording of Theorem 3.