MATH 532/736I, LECTURE 2

1. Finish Previous Notes

2. Assignment: Problems 1, 2, 8, 9 and 10 from Homework 1.

3. **Components of an Axiomatic Systems:**

   (i) Undefined Terms (points, lines)

   (ii) Defined Terms (parallel)

   (iii) Axioms

   (iv) A system of logic (if A or B is true and if not A is true, then B is true)

   (v) Theorems

4. **Definition 1:** An axiomatic system is *consistent* if there is not in the system any two axioms, any axiom and theorem, or any two theorems that contradict each other.

   Comment: Start example below.

5. **Definition 2:** An axiom in an axiomatic system is *independent* if it cannot be proved from the other axioms. If each axiom in the axiomatic system is independent, then the axiomatic system is said to be *independent*.

   Comment: Continue example below.

6. **Definition 3:** An axiomatic system is *complete* if every statement containing undefined or defined terms of the system can be proved valid or invalid.

   Comments: A complete axiomatic system uniquely determines what the system is. There is a unique "model" up to isomorphisms that is described by the axiomatic system. It is impossible to add a new independent axiom to a complete axiomatic system.

   Finish example.

7. **Example:**

   Axiom 1: There exist exactly 4 points.

   Axiom 2: Given any two distinct points, there is exactly one line that they lie on.

   Axiom 3: Given any line, there are exactly 2 points on it.

   Questions:

   - What might be defined terms here?
   - What might be undefined terms here?

- Is the axiomatic system consistent? How do we show this? (What if the answer were different?)
- Are the axioms independent? How do we justify our answer? (What if the answer were different?)
- Is this axiomatic system complete? How do we justify this? (What if the answer were different?) Describe incidence tables and isomorphisms.