1. Hand out and go over syllabus.

2. Class photos.

3. No homework (today).

4. Logic in the Last Century:
   - Are there statements that can be made in mathematics which are true but which we cannot prove?
     Remark 1: The answer cannot be “Yes” since if there exist such statements, we would not know they are true, so we would not know they exist.
     Remark 2: Remark 1 is wrong. Such statements are known to exist (but that does not mean that we know what they are).
   - Is mathematics consistent? Is it possible that some day a proof will exist that $1 + 1 = 3$?
     Remark 1: If mathematics were not consistent, we wouldn’t have this class. Therefore, mathematics must be consistent.
     Remark 2: Remark 1 is questionable. No proof exists that mathematics is consistent. It is known, however, that if mathematics is consistent, then we cannot prove it is consistent.

5. Back to Euclid’s Time:
   (i) Constructions:
     - Find the perpendicular bisector of a line segment.
     - Duplicate an angle.
     - Divide a segment into 3 equal pieces.
     - Given a line $\ell$ and a point $P$ not on $\ell$, construct a line $\ell'$ parallel to $\ell$ that passes through $P$.
     - Find the center of a given circle.
     - Given a line $\overrightarrow{DE}$ and two points $A$ and $B$ on one side of the line, find $C$ on $\overrightarrow{DE}$ so that $\angle ACD = \angle BCE$.
     - Given a circle $C$ and a point $P$ outside $C$, construct a line $\ell$ tangent to $C$ that passes through $P$.
     - Given a circle $C$ and two points $P$ and $Q$ outside $C$, construct the circles that are tangent to $C$ and pass through $P$ and $Q$. 
(ii) Even More Basic Constructions:

– Given two points, construct a line through them. (What if the points are far apart and the straightedge and compass are small by comparison?)
– Draw a circle with a given radius and center.

(iii) Euclid’s Axioms (or Postulates):

1. A straight line can be drawn through any two points.
2. A straight line can be extended in either direction as long as we wish.
3. Given any point \( P \) and any distance \( r \), we can draw a circle of radius \( r \) centered at \( P \).
4. All right angles are equal.
5. If a line \( \ell \) intersects two lines \( \ell_1 \) and \( \ell_2 \) and makes two interior angles on the same side of \( \ell \) each less than a right angle, then the lines \( \ell_1 \) and \( \ell_2 \) intersect on that side of \( \ell \).

Comment: From these axioms, Euclid developed theorems (or propositions).

6. Between Euclid and a Century Ago:

- What if the axioms were different? Consider something like:
  1. There exist 3 non-collinear points.
  2. Given two points, there exists a line passing through them.
  3. Any two distinct lines intersect in exactly two points.

Do these axioms make sense? Are they simply describing something that’s not true and therefore nonsense?

- Describe great circles on a sphere, and explain why the axioms above all hold in this context.