Answers to Test 1, 1999

1. (a) \((2, 3, 1)\)
   (b) \(\sqrt{14}\)
   (c) \(\pi/3\)
   (d) \(7\sqrt{3}/2\)
   (e) \(S = (5, -1, -4)\) or \(S = (9, 5, -2)\) or \(S = (3, 3, 2)\)

2. (a) \((6, 3t^2, 6t)\)
   (b) \((1, 0, 0)\)
   (c) 7

3. \(2x - y + z = -5\)

4. (a) If there is a \(P = (x, y, z)\) on \(\ell_1\) and \(\ell_2\), then there is some \(t\) and some \(s\) such that
   \[
   (x, y, z) = (2 + t, 0, -1 + t) = (3, 2s, 1 + s).
   \]
   Since \(2 + t = 3\), \(t = 1\). Since \(0 = 2s\), \(s = 0\). But then \(-1 + t = 0\) and \(1 + s = 1\)
   so that \(-1 + t \neq 1 + s\). This implies that \(P\) cannot exist. In other words, \(\ell_1\) and \(\ell_2\) do not intersect. Note now that \((1, 0, 1)\) is a vector parallel to \(\ell_1\) and \((0, 2, 1)\) is a vector parallel to \(\ell_2\). Also, \((1, 0, 1) \neq c(0, 2, 1)\) for any number \(c\) (otherwise, \(1 = c \times 0\), which is impossible). Hence, \(\ell_1\) and \(\ell_2\) are not parallel.
   (b) \(2/3\)

5. (i) (d), \((0, 0, 1/2)\) (or \((0, 0, -1/2)\))
   (ii) (b), a hyperbola
   (iii) (a), \((0, 0, 0)\)