1. Calculate an equation for the tangent plane to the surface

\[ 2(x - 2)^2 + (y - 1)^2 + (z - 3)^2 = 10 \]

at the point \((3, 3, 5)\).

Equation of tangent plane \(P\):

\[ x + y + z = 11 \]

Solution: Let \(F(x, y, z) = 2(x - 2)^2 + (y - 1)^2 + (z - 3)^2\) (or \(F(x, y, z) = 2(x - 2)^2 + (y - 1)^2 + (z - 3)^2 - 10\)). Then \(\nabla F = \langle 4(x - 2), 2(y - 1), 2(z - 3) \rangle\). Hence, \(\nabla F(3, 3, 5) = \langle 4, 4, 4 \rangle = 4\langle 1, 1, 1 \rangle\). Thus, \(\langle 1, 1, 1 \rangle\) is normal to the tangent plane and \((3, 3, 5)\) is a point on the plane, and we deduce that the equation for the tangent plane is \(x + y + z = 11\). □

2. Let \(f(x, y) = x^2 - y^2 + 1\), and let \(P\) be the point \((0, 1)\). There are infinitely many different values for the directional derivative of \(f(x, y)\) at the point \(P\) (since there are infinitely many directions that can be used to compute the directional derivative). Which of these is minimal? In other words, what is the least value of the directional derivative of \(f(x, y)\) at the point \(P\)?

Least value of directional derivative at \(P\):

\[ -2 \]

Solution: Here, \(\nabla f = \langle 2x, -2y \rangle\) so that \(\nabla f(P) = \langle 0, -2 \rangle\) (recalling \(P = (0, 1)\)). The directional derivative is minimized at \(P\) when one goes in the direction of \(-\nabla f(P)\). One can now compute the directional derivative in this direction. On the other hand, we also know that the minimal value of the directional derivative when we do this is \(-\|\nabla f(P)\|\). Since \(\|\nabla f(P)\| = \sqrt{0^2 + (-2)^2} = \sqrt{4} = 2\), the answer is \(-2\). □