Calculate the arc length of the curve traced by

\[ x = t^3 - 3t, \quad y = \frac{4t^3}{3} + 4t, \quad z = 3t^2 \]

where \(0 \leq t \leq 3\). Simplify your answer.

Arc Length: [60] (Simplify)

**Solution:** First, we calculate

\[ x'(t) = 3t^2 - 3, \quad y'(t) = 4t^2 + 4, \quad \text{and} \quad z'(t) = 6t. \]

Next, we calculate

\[ x'(t)^2 + y'(t)^2 + z'(t)^2 = (3t^2 - 3)^2 + (4t^2 + 4)^2 + (6t)^2 \]
\[ = (9t^4 - 18t^2 + 9) + (16t^4 + 32t^2 + 16) + 36t^2 \]
\[ = 25t^4 + 50t^2 + 25 \]
\[ = (5(t^2 + 1))^2. \]

The arc length is given by

\[ \int_0^3 \sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2} \, dt = \int_0^3 5(t^2 + 1) \, dt = 5 \left( \frac{t^3}{3} + t \right)_0^3 = 5(9 + 3) = 60. \]