1. Calculate an equation for the plane that passes through the point \((2, 0, 1)\) and is parallel to the plane \(2x - y + 3z = -4\).

Equation for Plane: \(2x - y + 3z = 7\)

Solution: The vector \(\langle 2, -1, 3 \rangle\) is normal to the plane \(2x - y + 3z = -4\) and, therefore, will be normal to the parallel plane. Hence, an equation for the parallel plane is \(2x - y + 3z = d\) for some number \(d\). Since the parallel plane passes through \((2, 0, 1)\), we see that \(d = 2 \cdot 2 - 0 + 3 \cdot 1 = 7\). Thus, \(2x - y + 3z = 7\) is an equation for the parallel plane. ■

2. The plane \(\mathcal{P}\) consists of the points \((x, y, z)\) satisfying \(x - y + z = 0\). The point \(A = (1, 2, 3)\) is not on the plane \(\mathcal{P}\). Find a point \(B\) on the plane \(\mathcal{P}\) such that the distance from \(A\) to \(B\) is minimal. In other words, what is the point \(B\) on the plane \(\mathcal{P}\) that is nearest to the point \(A\)?

\[ B = \left( \frac{1}{3}, \frac{8}{3}, \frac{7}{3} \right) \]

Solution 1: The line \(\overrightarrow{AB}\) through \(A\) and \(B\) is perpendicular to the plane \(\mathcal{P}\), so the normal \(\overrightarrow{n} = \langle 1, -1, 1 \rangle\) to \(\mathcal{P}\) is parallel (in the direction of) \(\overrightarrow{AB}\). Since \(A = (1, 2, 3)\) is on \(\overrightarrow{AB}\), the points on line \(\overrightarrow{AB}\) satisfy the parametric equations \(x = 1 + t, y = 2 - t\) and \(z = 3 + t\). The point \(B\) is the intersection of this line with the plane \(\mathcal{P}\). Since \(\mathcal{P}\) is given by the equation \(x - y + z = 0\), the intersection of \(\overrightarrow{AB}\) with \(\mathcal{P}\) is determined by the value of \(t\) satisfying \((1 + t) - (2 - t) + (3 + t) = 0\). This simplifies to \(2 + 3t = 0\) or \(t = -2/3\). Hence, the coordinates of \(B\) are given by \(x = 1 + (-2/3) = 1/3, y = 2 - (-2/3) = 8/3\) and \(z = 3 + (-2/3) = 7/3\). In other words, \(B = \left( \frac{1}{3}, \frac{8}{3}, \frac{7}{3} \right)\). ■

Solution 2: The vector \(\overrightarrow{n} = \langle 1, -1, 1 \rangle\) is normal to the plane \(\mathcal{P}\). The point \(C = (0, 0, 0)\) satisfies the equation \(x - y + z = 0\) and so is on \(\mathcal{P}\). Observe that \(\overrightarrow{AC} = \langle -1, -2, -3 \rangle\), and

\[ \overrightarrow{AB} = \text{proj}_\overrightarrow{n} \overrightarrow{AC} = \frac{\overrightarrow{n} \cdot \overrightarrow{AC}}{\|\overrightarrow{n}\|^2} \overrightarrow{n} = \frac{-2}{3} \cdot \langle 1, -1, 1 \rangle = \frac{-2}{3} \cdot \overrightarrow{\langle 1, 2, -2 \rangle}. \]

We can view \(\overrightarrow{AB}\) as telling us how much each coordinate of \(A\) has to change to go from \(A\) to \(B\). Thus, \(B = (1 + (-2/3), 2 + (2/3), 3 + (-2/3)) = \left( \frac{1}{3}, \frac{8}{3}, \frac{7}{3} \right)\). ■