1. Find parametric equations for the line $\ell$ parallel to the line given by

$$x = -1 + 2t, \quad y = 2 - t, \quad z = 1 + t$$

and passing through the point $(-2, 2, 3)$.

**Parametric Equations for $\ell$:**

- $x = -2 + 2t$
- $y = 2 - t$
- $z = 3 + t$

**Solution**: Since the given line is parallel to (going in the direction of) the vector $\vec{v} = (2, -1, 1)$, the line $\ell$ must be as well. Also, $\ell$ passes through the point $(-2, 2, 3)$. The answer follows. ■

2. Let $\ell$ be the line given by the parametric equations $x = 3t, y = 1 - 2t$ and $z = 1 + 2t$. Let $\ell'$ be the line given by the parametric equations $x = 2 - t, y = 2 + t$ and $z = -t$. The lines $\ell$ and $\ell'$ intersect at a point $P$. Calculate the point $P$. Show work and simplify your answer.

$P = (9, -5, 7)$ (simplify)

**Solution**: Since the common point $P$ can occur for different values of $t$ (one for $\ell$ and a different one for $\ell'$), we change the names of the parameters so that they are not the same and replace $t$ in the line $\ell'$ with $s$. Then, setting the different coordinates equal, we want to know for what $t$ and $s$ we have

$$3t = 2 - s$$
$$1 - 2t = 2 + s$$
$$1 + 2t = -s$$

Adding the first two of these equations, we see that $1 + t = 4$ or $t = 3$. Plugging in $t = 3$ into the first equation, we obtain $9 = 2 - s$ so that $s = -7$. We only need to know $t$ if we know the two lines intersect at a point, but the value of $s$ helps us check our answer. Plugging in $t = 3$ into the equations for $\ell$ gives $P = (9, -5, 7)$. Plugging in $s = -7$ into the equations for $\ell'$ (with $t$ replaced by $s$) also gives $P = (9, -5, 7)$. Thus, the two lines $\ell$ and $\ell'$ intersect at $(9, -5, 7)$. ■

*These problems were taken directly from the homework (with no number changes) that you did at home and we did in class. You’re welcome.*