1. Let \( A = (2, 1, -3) \), and let \( P \) be the plane given by \( x + y - z = 0 \). Calculate the point \( B \) on the plane \( P \) that is nearest to \( A \). Simplify your answer.

Point \( B \): \((0, -1, -1)\)

**Solution 1:** First, we find parametric equations for a line \( \ell \) perpendicular to the plane \( P \) that passes through \( A \). Since a normal to the plane is \( \langle 1, 1, -1 \rangle \), this vector is parallel to (in the direction of) \( \ell \). Since \( \ell \) goes through \( A \), parametric equations for \( \ell \) are given by \( x = 2 + t \), \( y = 1 + t \) and \( z = -3 - t \). The point \( B \) is the point \((2 + t, 1 + t, -3 - t)\) on \( \ell \) which is also on \( P \). Since \( P \) is given by \( x + y - z = 0 \), we want 

\[
(2 + t) + (1 + t) - (-3 - t) = 0 \quad \text{or, equivalently,} \quad 6 + 3t = 0.
\]

This implies \( t = -2 \), so the point \( B \) is \((2 - 2, 1 - 2, -3 - (-2)) = (0, -1, -1)\). ■

**Solution 2:** The point \( Q = (0, 0, 0) \) is on the plane \( P \) (any point \( Q \) on \( P \) can be used here). We compute the projection of the vector \( \overrightarrow{QA} = \langle 2, 1, -3 \rangle \) onto the normal \( \overrightarrow{n} = \langle 1, 1, -1 \rangle \) to plane \( P \). This is given by

\[
\text{proj}_{\overrightarrow{n}} \overrightarrow{QA} = \frac{\overrightarrow{n} \cdot \overrightarrow{QA}}{\|\overrightarrow{n}\|^2} \overrightarrow{n} = \frac{6}{\sqrt{3}} \langle 1, 1, -1 \rangle = 2 \langle 1, 1, -1 \rangle = \langle 2, 2, -2 \rangle.
\]

We want then a point \( B \) such that \( \overrightarrow{BA} = \langle 2, 2, -2 \rangle \). Since \( A = (2, 1, -3) \), we deduce \( B = (2 - 2, 1 - 2, -3 - (-2)) = (0, -1, -1) \). ■

2. The two planes given by \( x - 2y + z = 4 \) and \( 2x + y - 2z = 5 \) intersect in a line \( \ell \). Find the parametric equations for the line \( \ell' \) which is parallel to \( \ell \) and passes through the point \((1, 1, 0)\).

Line:

\[
\begin{align*}
x &= 1 + 3t \\
y &= 1 + 4t \\
z &= 5t
\end{align*}
\]

**Solution:** The normals to the planes, given by \( \overrightarrow{n}_1 = \langle 1, -2, 1 \rangle \) and \( \overrightarrow{n}_2 = \langle 2, 1, -2 \rangle \), are both perpendicular to a vector parallel to \( \ell \) and, hence, parallel to \( \ell' \). So a vector perpendicular to both \( \overrightarrow{n}_1 \) and \( \overrightarrow{n}_2 \) will be parallel to \( \ell' \). We can find such a vector by computing

\[
\overrightarrow{n}_1 \times \overrightarrow{n}_2 = \begin{vmatrix}
\hat{i} & \hat{j} & \hat{k} \\
1 & -2 & 1 \\
2 & 1 & -2
\end{vmatrix} = \langle 3, 4, 5 \rangle.
\]

Since \((1, 1, 0)\) is on \( \ell' \), parametric equations for \( \ell' \) are given by \( x = 1 + 3t \), \( y = 1 + 4t \) and \( z = 5t \). ■