1. Complete the summation notation below.

\[ 1 + 3 + 3^2 + 3^3 + \cdots + 3^{99} + 3^{100} = \sum_{k=1}^{\square} \square \]

2. Complete the summation notation below.

\[ 1 - 3 + 3^2 - 3^3 + \cdots - 3^{99} + 3^{100} = \sum_{k=1}^{\square} \square \]

3. Evaluate

\[ \sum_{k=1}^{20} \left( \frac{1}{k+1} - \frac{1}{k+2} \right). \]

4. What is the value of the sum

\[ 20 + 22 + 24 + 26 + \cdots + 98 + 100 \]?

5. What is the value of

\[ 2^{10} - 2^8 + 2^6 - 2^4 + 2^2 - 1 \]?

Express your answer in the form \( \frac{1}{a} (2^b - c) \) for some integers \( a, b, \) and \( c \).

6. Which if any of \( (A \cap B) \cup (A \cap C) \) and \( (A \cup B) \cap (A \cup C) \) is the same as \( A \cap (B \cup C) \)?

7. What are DeMorgan’s Laws for sets?

8. True or False (justify your answer). If \( A \cup C = B \cup C \), then \( A = B \).

9. True or False (justify your answer). If \( A \cap C = B \cap C \), then \( A = B \).

10. If \( A = \{a, b, c\}, B = \{a, c, e, f\}, C = \{c, d, f, g\} \), and \( U \) is a universal set containing \( A, B, \) and \( C \), then what are the following:

\( a) \quad (A \cap B) - C \quad b) \quad (B \cup C) \cap (A - C) \)

11. If \( A = \{1, 2\} \), then what is \( A \times A \)?

12. Do the sets \( A = \{n \in \mathbb{Z} : n \mod 2 = 1\}, B = \{n \in \mathbb{Z} : n \mod 4 = 0\}, \) and \( C = \{n \in \mathbb{Z} : n \mod 4 = 2\} \) partition the set \( \mathbb{Z} \)?

13. What is the power set of \( \{a, b, c\} \)? (Indicate its elements.)

14. Kelly reads a chapter of a book beginning at the top of page 80 and ending at the bottom of page 100. How many pages has Kelly read in this chapter?

15. What is the 20th element in the array


16. How many different ways can the letters in THREE be arranged in a row?

17. How many different ways can the letters in THREE be arranged in a row if the two E’s are not to be next to each other?

18. There are 5 women and 4 men wanting to form a committee using 2 of the women and 2 of the men. How many different committees are possible? Simplify your answer.

19. If the product \( (3x - 2)^{11} (x - 2)^{15} (12x^3 - 10x^2 + 2x + 5) \) is expanded, the result is a 29th degree polynomial. In other words, the expanded product is of the form

\[ a_{29}x^{29} + a_{28}x^{28} + \cdots + a_1x + a_0. \]

What is the sum of the coefficients \( a_{29}, a_{28}, \ldots, a_1 \) and \( a_0 \)?
20. The 10th row of Pascal’s triangle is

\[ 1 \quad 10 \quad 45 \quad 120 \quad 210 \quad 252 \quad 210 \quad 120 \quad 45 \quad 10 \quad 1. \]

What is the 11th row?

21. What is the coefficient of \( x^5 \) in the expansion of \((2x - 1)^8\)?

22. A chessboard is an 8 \times 8 board. It contains 64 small squares, but it also contains some larger squares.
   (a) How many 2 \times 2 squares are on a chessboard?
   (b) How many 3 \times 3 squares are on a chessboard?
   (c) How many 7 \times 7 squares are on a chessboard?
   (d) How many total squares are on a chessboard? Express your answer using summation notation and evaluate the sum.

23. Complete the following proof that

\[
\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \cdots + \frac{1}{\sqrt{n-1}} + \frac{1}{\sqrt{n}} \geq \sqrt{n}.
\]

Let \( P(n) \) be the statement that

\[
\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \cdots + \frac{1}{\sqrt{n-1}} + \frac{1}{\sqrt{n}} \geq \boxed{}.
\]

We prove that \( P(n) \) is true for every positive integer \( n \) by using \boxed{\text{induction}}. We first show that \boxed{\text{is true}}. Since \( \sqrt{1} = 1 \), we see that \( 1/\sqrt{1} \geq \sqrt{1} \). Thus, \boxed{\text{is in fact true}}. Next, we suppose that \( k \) is an integer \( \geq 1 \) such that \boxed{\text{is true}}. This is called our induction hypothesis. Thus, our induction hypothesis is asserting that

\[
\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \cdots + \frac{1}{\sqrt{k-1}} + \frac{1}{\sqrt{k}} \geq \boxed{\text{}}.
\]

From the induction hypothesis, we obtain that

\[
\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \cdots + \frac{1}{\sqrt{k-1}} + \frac{1}{\sqrt{k}} + \frac{1}{\sqrt{k+1}} \geq \sqrt{k} + \frac{1}{\sqrt{k+1}}.
\]

Observe that \( \sqrt{k(k+1)} > \sqrt{k^2} = k \) so that

\[
\sqrt{k(k+1)} + 1 > k + 1.
\]

Dividing by \( \sqrt{k+1} \), we deduce that

\[
\boxed{\text{}} + \frac{1}{\sqrt{k+1}} > \boxed{\text{}}.
\]

We deduce that

\[
\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \cdots + \frac{1}{\sqrt{k-1}} + \frac{1}{\sqrt{k}} + \frac{1}{\sqrt{k+1}} \geq \boxed{\text{}}.
\]

We have shown that if \( P(k) \) is true, then \boxed{\text{is true}}. This completes the induction argument. Therefore, \( P(n) \) is true for every integer \( n \geq 1 \). \( \blacksquare \)