1. | p | q | p ∧ q | ~ p | ~ q | p ∨ ~ q | ~ p ∨ q | ~ p ∨ (p ∧ q) | ~ q ∨ (p ∧ q) | (p ∨ ~ q) ∨ (~ p ∨ q) |
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2. p ∨ ~ q ≡ ~ q ∨ (p ∧ q) and ~ p ∨ q ≡ ~ p ∨ (p ∧ q)

3. (p ∨ ~ q) ∨ (~ p ∨ q)

4. There are no contradictions.

5. ~ p ∧ ~ q

6. ~ p ∨ ~ q

7. There are two variables, p and q, so the truth table would have 4 rows (like above). Thus, in each column, there are 4 rows of spaces each of which is either filled with “T” or “F”. There are 16 different ways to fill the 4 spaces with T’s and F’s. It follows that if there are 17 or more columns (each representing a statement form), at least two of the columns must have the 4 spaces filled in exactly the same way. In other words, at least two of the statement forms in a collection of 17 or more forms would have to be equivalent.

8. if q then p (that is, q → p)

9. if not p then not q (that is, ~ p → ~ q)

10. p and not q (that is, p ∧ ~ q)

11. if not q then not p (that is, ~ q → ~ p)

12. the contrapositive

13. | p | q | p ∧ q | ~ q | ~ p → p ∧ q |
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14. The first two arguments are valid (the second two are invalid).

15. ∃x ∈ D such that ~ P(x)

16. ∀x ∈ D, ~ P(x)

17. ∃ positive integers n such that ∀ integers a, b, c, and d, n ≠ a² + b² + c² + d²

18. ∃a ∈ Q and ∃b ∈ Q such that ∀c ∈ Q, c ≤ a or c ≥ b.

19. ~ Q(x) ⇒ P(x) and P(x) ⇔ ~ Q(x)

20. For x = 0, if 8x + 13y = 1 then y = 1/13 so no integer y satisfies 8x + 13y = 1 (in this case).
   For x = 1, if 8x + 13y = 1 then y = -7/13 so no integer y satisfies 8x + 13y = 1 (in this case).
   For x = 2, if 8x + 13y = 1 then y = -15/13 so no integer y satisfies 8x + 13y = 1 (in this case).
   For x = 3, if 8x + 13y = 1 then y = -23/13 so no integer y satisfies 8x + 13y = 1 (in this case).
   For x = 4, if 8x + 13y = 1 then y = -31/13 so no integer y satisfies 8x + 13y = 1 (in this case).
   Thus, by the method of exhaustion, ∀x ∈ {0, 1, 2, 3, 4}, there does not exist an integer y such that 8x + 13y = 1.

21. Since 8 × 5 + 13 × (-3) = 1, there exist x and y such that 8x + 13y = 1 (namely, x = 5 and y = -3 work).

22. 1, 2, 3, 4, 6, and 12

23. 3, 5, and 13
24. $2^4 \cdot 5$.

25. Yes. Since $9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 45$ and 45 is divisible by 3, the number 987654321 is divisible by 3.

26. Since $200 \mod 7 = 4$, it will be the same as four days from today. If you are reading this on Thursday, the answer is “Monday”.

27. Yes. To check if 241 is prime, we need only see if 241 is divisible by a prime $\leq \sqrt{241}$. Since $16^2 = 256$, we need only consider divisibility by primes $< 16$. One checks directly that 241 is not divisible by 2, 3, 5, 7, 11, and 13, which then justifies that 241 is prime.

28. $73 \mod 5 = 3$, $-73 \mod 5 = 2$, $29 \mod 4 = 1$, $-29 \mod 4 = 3$

29. $[3.6] = 3$, $\lceil 3.6 \rceil = 4$, $[-1.9] = -2$, $\lceil -1.9 \rceil = -1$

30. Assume $\sqrt{2}$ is rational. Then there exist integers $a$ and $b$ with $b \neq 0$, with $\sqrt{2} = a/b$, and with $a/b$ reduced (so that $a$ and $b$ have no common prime factors). Since $\sqrt{2} = a/b$, we obtain

$$b\sqrt{2} = a$$

so that

$$2b^2 = a^2.$$ 

We deduce that $a$ is even. Therefore, there is an integer $k$ such that $a = 2k$. Substituting this into $2b^2 = a^2$, we obtain

$$2b^2 = (2k)^2 = 4k^2$$

so that $b^2 = 2k^2$. We deduce that $b$ is even. This is a contradiction since $a/b$ is reduced and $a$ and $b$ are even. Therefore, our assumption is wrong and $\sqrt{2}$ is irrational. ■