1. Go over homework questions.

2. Homework: pages 124–125, numbers 1, 3, 6, 7, 9, 15, 21, 25, 28, 29 (some from last time)
   Read from Example 3.1.5 "Proving a Theorem" on page 117 to the middle of page 120
   Quiz: Thursday (09/20)
   Test: Thursday (09/27)

3. Definitions: $n \in \mathbb{Z}$ is even $\iff \exists k \in \mathbb{Z}$ such that $n = 2k$
   $n \in \mathbb{Z}$ is odd $\iff \exists k \in \mathbb{Z}$ such that $n = 2k + 1$

4. Definitions:
   $n \in \mathbb{Z}$ with $n > 1$ is prime $\iff (\forall$ positive integers $r$ and $s, n = rs \implies$ either $r = 1$ or $s = 1)$
   $n \in \mathbb{Z}$ with $n > 1$ is composite $\iff \exists$ integers $r > 1$ and $s > 1$ such that $n = rs$

5. Constructive and Nonconstructive Proofs of Existence (for existential statements)

   Examples: (1) There exist integers $x$ and $y$ such that $5x + 8y = 1$.
   (2) There exist numbers that are not rational. (Use $\sqrt{2}$ and $0.1010010001\ldots$)
   (3) There exist irrational numbers $a$ and $b$ such that $a^b$ is rational.

6. The Method of Exhaustion (for universal statements)

   Examples: (1) The number 6174.
   (2) Every even number $n$ with $4 \leq n \leq 30$ can be written as a sum of two primes.
   (3) Every even number $n \geq 4$ can be written as a sum of two primes.

7. Theorem 3.1.1 If the sum of two integers is even, then so is their difference.

   What does “two” mean here?

   Give the proof (note related to reading assignment).

8. Common Errors: (1) Arguing from examples (illustrate with 6174 and Theorem 3.1.1).
   (2) Using “if” inappropriately (Suppose $\sqrt{2}$ is rational. If $\sqrt{2}$ is rational, then $\sqrt{2} = a/b$ for some integers $a$ and $b$.)

   Note that the students may want to look over other common errors on page 121.

9. Disproving a universal statement with a counterexample

   Example: pages 124, number 16

10. Miscellaneous Examples: page 125, numbers 26, 37

11. Definitions: $r$ is rational $\iff \exists$ integers $a$ and $b$ such that $r = a/b$ and $b \neq 0$
   $r \in \mathbb{R}$ is irrational $\iff r$ is not rational

12. Example: Explain why the sum of two rational numbers is rational.