1. Return quizzes (35 total, 88% ave., 18 perfects; 21 A’s, 6 B’s, 3 C’s, 2 D’s, 3 F’s)

2. Go over homework questions.

3. Homework: pages 87–88, numbers 1 (a-d), 4, 5, 6, 7, 9, 11 (a,c), 20 (a,c), 21 (a,b), 29, 32
   Quiz: Thursday (09/06)

4. **Contradiction Rule:** If you can show that the supposition that \( p \) is false leads logically to a contradiction, then you can conclude that \( p \) is true.

5. **Examples:**
   (1) \( \sqrt{2} \) is irrational
   (2) there exist irrational numbers \( \alpha \) and \( \beta \) such that \( \alpha \beta \) is rational

6. **Definition:** A *predicate* is a sentence that contains a finite number of variables and becomes a statement when specific values are substituted for the variables. The *domain* of a predicate variable is the set of all values that may be substituted for the variable.

7. **Definition:** If \( P(x) \) is a predicate and \( x \) has domain \( D \), the *truth set* of \( P(x) \) is the set of elements of \( D \) that make \( P(x) \) true when substituted for \( x \).

   **Note:** This can be written \( \{ x \in D | P(x) \} \) (the set of all \( x \) in \( D \) such that \( P(x) \) holds).

8. **Notation:** Let \( P(x) \) and \( Q(x) \) be predicates with domain \( D \). The notation \( P(x) \implies Q(x) \) means every element of the truth set of \( P(x) \) is in the truth set of \( Q(x) \). The notation \( P(x) \iff Q(x) \) means \( P(x) \) and \( Q(x) \) have identical truth sets.

9. **Example:**
   (1) Suppose \( P(x) \) is “\( x \) is even” and \( Q(x) \) is “\( x \) is a power of 2” and \( D = \{ 2, 3, \ldots, 100 \} \).
   (2) Suppose further that \( R(x) \) is “\( x > 200 \)” (same \( D \)).
   (3) What if \( D = \{ 2, 3, 4 \} \)?

10. **Definition and Notation:** The symbol “\( \forall \)” denotes “for all” and the symbol “\( \exists \)” denotes “there exists”. They are called *quantifiers*. The first is a *universal quantifier* and the second an *existential quantifier*.

11. \( \forall x \in D, Q(x) \) (a *universal statement*)
    \( \exists x \in D \) such that \( Q(x) \) (an *existential statement*)

    When is each true?
    When is each false? (Note what a *counterexample* is.)

12. **Examples:**
    (1) Let \( D = \{ 1, 2, \ldots, 10 \} \). Then \( \forall x \in D, x < 20 \).
    (2) Let \( D = \{ 1, 2, \ldots, 10 \} \). Then \( \forall x \in D, x > 5 \).
    (3) Also, \( \exists x \in D \) such that \( x^2 \in D \) (same \( D \)).

13. Set Notations (\( \mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C} \) or \( \mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C} \))

    **Examples:**
    (1) \( \exists x \in \mathbb{Z} \) such that \( x^2 = x \) (write in English)
    (2) \( \forall x \in \mathbb{R}, x^2 > x \) (not true)

14. **Negations:**
    \( \sim (\forall x \in D, Q(x)) \equiv \exists x \in D \) such that \( \sim Q(x) \)
    \( \sim (\exists x \in D \) such that \( Q(x)) \equiv \forall x \in D, \sim Q(x) \)

    **Note:** The negation of a universal statement is logically equivalent to an existential statement. The negation of an existential statement is logically equivalent to a universal statement.

    **Examples:** pages 87–88, numbers 11 (b,d), 20 (b,d)