1. Return quizzes (25 total, 13.28% ave.; 23 A’s, 2 B’s)

2. Go over homework.

3. Homework: page 438, numbers 1, 3, 5
page 451, numbers 3, 5, 10
Solve $u_0 = 4$, $u_1 = -1$, and $u_{n+1} = u_n + 2u_{n-1}$ for $n \geq 1$.

4. Recursion Examples:
   (1) The Tower of Hanoi (show that $a_{n+1} = 2a_n + 1$ and $a_n = 2^n - 1$)
   (2) Maximal number of regions divided by $n$ lines (show $a_{n+1} = a_n + n + 1$)
   (3) Fibonacci Numbers ($f_1 = 1$, $f_2 = 1$, and $f_{n+1} = f_n + f_{n-1}$ for $n \geq 2$)

5. Solving Linear Recurrence Relations: Given $u_0$, $u_1$, and $u_{n+1} = au_n + bu_{n-1}$ for $n \geq 1$, there exist numbers $A$ and $B$ such that
   $$u_n = A\alpha^n + B\beta^n$$
   where $\alpha$ and $\beta$ are the roots of $x^2 - ax - b$ (provided $\alpha \neq \beta$). The numbers $A$ and $B$ are determined by the values of $u_0$ and $u_1$.

6. Examples:
   (1) Solve $u_0 = 1$, $u_1 = 3$, and $u_{n+1} = 3u_n - 2u_{n-1}$ for $n \geq 1$.
   (2) Solve $u_0 = 3$, $u_1 = 1$, and $u_{n+1} = 2u_n + 3u_{n-1}$ for $n \geq 1$.
   (3) Solve for $f_n$ (the $n^{th}$ Fibonacci number).

7. Other Recursion Examples:
   (1) page 438, number 2
   (2) page 438, number 4
   (3) page 451, number 4
   (4) page 451, number 6

8. A Final Example: Solve the linear recurrence $u_0 = 6$, $u_1 = 1$, $u_2 = 9$, and $u_{n+1} = 2u_n + u_{n-1} - 2u_{n-2}$ for $n \geq 2$. The sequence begins
   $$6, 1, 9, 7, 21, 31, 69, 127, 261, 511, 1029, 2047.$$