1. Go over homework questions.

2. Homework: pages 242–243, numbers 1, 2, 5, 6, 7, 8, 11(a), 13, 15(a,c,e,f), 17(a,d), 18(a)

Quiz: Thursday (10/11)

3. Sets and Elements of Sets: Strictly speaking sets and elements are undefined terms. Think of a set as a collection of its elements.

4. Notation via Examples:
   (1) \{-1, 2, 5\} is the set consisting of the elements \(-1, 2,\) and 5
   (2) \{Bill, giraffe, book\} is the set consisting of the elements Bill, giraffe, and book
   (3) \{\} is the set consisting of no elements and is called the empty set (also denoted \(\emptyset\))
   (4) \{1, \{1\}\} is the set consisting of the two elements 1 and the set \{1\}
   (5) \{x \in \mathbb{Z} \mid 1 \leq x \leq 10\} (the set of \(x\) in \(\mathbb{Z}\) such that \ldots) is the set consisting of the integers from 1 to 10
   (6) What is \(\{x \mid x \in \mathbb{Z}, 1 \leq x \leq 10\}\)?
   (7) What is \(\{x \in \mathbb{Z} \mid 2 \leq x^4 \leq 10\}\)?

5. Definition and Notation: For two sets \(A\) and \(B\), the notation \(A \subseteq B\) means \(\forall x, \text{if } x \in A, \text{ then } x \in B\). The notation \(A \subseteq B\) can be read in any of the following ways:
   - \(A\) is a subset of \(B\)
   - \(A\) is contained in \(B\)
   - \(B\) contains \(A\)

6. Question: What would \(A\) is not a subset of \(B\) mean?

   Answer: \(A \nsubseteq B \iff \exists x \in A \text{ such that } x \notin B\)

7. Definition: For two sets \(A\) and \(B\), we say \(A\) is a proper subset of \(B\) if \(A \subseteq B\) and there is at least one element of \(B\) not in \(A\).

8. Definition: Two sets \(A\) and \(B\) are equal if \(A \subseteq B\) and \(B \subseteq A\).

9. Definitions and Notations: Let \(A\) and \(B\) be subsets of a “universal” set \(U\). Then

   \(A \cup B = \{x \mid x \in A \text{ or } x \in B\}\) ("A union \(B\")
   \(A \cap B = \{x \mid x \in A \text{ and } x \in B\}\) ("A intersect \(B\")
   \(A - B = \{x \mid x \in A \text{ and } x \notin B\}\) ("A minus \(B\")
   \(A^c = \{x \in U \mid x \notin A\}\) ("A complement")
   \(A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}\) ("Cartesian product of \(A\) and \(B\")

   Comments: Ordered tuples \((a, b)\) as above are like points. Two tuples \((a, b)\) and \((c, d)\) are equal if and only if \(a = c\) and \(b = d\). Also, the notions of simultaneously taking the union, intersection, or Cartesian product of more than two sets has a natural generalization of the above.

10. Examples:
    (1) \(\mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}\)
    (2) \(A \cap B \subseteq A \cup B\)
    (3) For \(A = \{x \in \mathbb{Z} \mid 1 \leq x \leq 4\}\) and \(B = \{x \in \mathbb{Z} \mid 3 \leq x \leq 7\}\), compute \(A \cup B, A \cap B, A - B, B - A, A \times B\) and \(A^c\) if \(U = \mathbb{Z}\).
    (4) page 243, number 9
    (5) page 243, number 15(b,d) (Venn diagrams)